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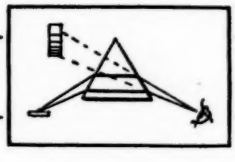

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VOLUME 30 OCTOBER, 1940 NUMBER 10

Editorial Comment

OPTICS FOR SKEPTICS

USERS of optical instruments in this country have little fear that the severe shortage which characterized periods during the 1914–1918 European war will be repeated during the current conflict. . . . While adjustment to the decline in imports from Europe has not yet been completed for a few materials, such as highly corrected photographic lenses for which there is relatively small demand, domestic production capacity and skill are reported as adequate to the task of supplying this country.

Although there had been some previous production, the American optical glass industry was created practically overnight in 1917 in response to the huge military demand for instruments. One of the greatest difficulties experienced in the first American production of optical glass was the procurement of proper pots for the melting of the raw materials. The iron content of pots which could be purchased on the open market was 100 times as great as the 0.02 percent maximum which the finished glass could tolerate without undue loss in transparency. The clay used in pots for the production of even commercial glass had been imported; and, although investigation revealed domestic clays of adequate quantity and variety, the knowledge of how to use them had to be acquired. The raw materials for the glass, including sand, potash and other substances, also had to be of a purity not then readily available. A country-wide search finally located satisfactory sand at Rockwood, Michigan; but an economical source of potash was not found until after the war. Determination of the proper proportions of the raw materials was complicated by the fact that during melting some of the ingredients are lost by volatilization, while others are added by solution from the pot. The father-to-son secrets of the European industry could not be revealed by the Allies even in wartime and the American research workers were forced to proceed as if in an unknown field, varying each component of the glass experimentally to determine its effect on the final product. The secrets were, however, learned and the United States Army was supplied almost entirely from domestic sources throughout the war.

Following the war, attention was turned from copying the European models to improving upon them. Many elements, including boron, barium and fluorine, were further investigated and their use extended. Through use of some of these elements it became possible to vary widely and independently the indexes of refraction and of dispersion which control the characteristics of optical glass. By proper arrangement of these new glasses, lens makers have been able to reduce chromatic aberration significantly.

During the past twenty years American producers have gained almost all the market for spectacle glass and manufacture the great majority of optical instruments. Some lens makers feel that domestic optical glass production does not afford the variety available in Europe, but American manufacturers are reportedly equipped to fill in whatever gaps may appear.—Quoted by permission from the *Industrial Bulletin of Arthur D. Little, Inc.*

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Reflectors Used in Highway Signs and Warning Signals. Parts I, II and III

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(Received August 2, 1940)

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I. Fundamental Optical Properties

The present paper is the first of a series relating to the selection and use of such (reflex) reflectors as give night-time visibility to highway signs and act as warning signals on trucks, etc. The optical function common to the various types of such reflectors—*retro-directive* reflection—is described, and the manner of its accomplishment by each of the two general types is explained. Although descriptions of the inherent characteristics of all retro-directive reflectors indicate the conditions of use for best results, variations between reflectors of different types and different makes create need for quantitative descriptions of performance in terms suitable for specifications. Chief among the terms proposed for this purpose is *specific intensity*, which designates the candlepower of the reflector per footcandle illumination under specified conditions of illumination and viewing. Other special terms required for describing these conditions are proposed. There are brief preliminary discussions of test methods and interpretation of test results.

REFLECTORS of a very special type are being ever more widely used to facilitate night travel on our highways and streets. Examples are the reflector buttons which stud the letters and symbols of STOP, CURVE, and other warning signs, and the red reflecting

plaques used as warning signals on the rear of trucks, busses, and bicycles. The value of such reflectors as safety devices is rapidly becoming recognized, and that aspect of the matter will be taken for granted. The purposes of the present paper are to describe the fundamental properties of reflectors of this general type, to show how an understanding of these properties can lead to their most effective use, and to lay down a general basis for selecting the particular type of reflector best suited for a given application.

A. Functional Description. Uses

Reflectors of the general type under consideration by no means "shine" in all directions when illuminated; rather, as shown in Fig. I-1, each one projects a narrow beam—a searchlight beam—directly back at the source (ordinarily a pair of automobile head lamps) which is furnishing it with light. The returning light beam is in the form of a narrow solid cone having the line from the source to the reflector as its axis and extending upward just enough to reach the eyes of the occupants of the automobile (Fig. I-2). But for the conservation achieved by confining the reflected beam to this narrow cone, the reflector would not appear as a bright source of light, for a reflector can utilize only that light which falls upon it. The same area of any other surface would receive as much light and the reflector appears tremendously brighter than a painted surface only because it returns substantially all this light into a small region near the source. The narrower the cone within which the return light is confined, the brighter will the reflector be, so long as the observer's eyes are within this cone.

The light is reflected back toward its source, whether or not that source is directly in front of the reflector. This essential function may well be contrasted with the type of reflection occurring at an ordinary plane mirror. Only if the light source is exactly on the perpendicular to the mirror is the light returned near to the source; the farther the source is moved to one side of this perpendicular, the farther the reflected light moves to the opposite side, as is illustrated in Fig. I-3. Reflectors which return light into the immediate neighborhood of its source, regardless of the position of that source, may appropriately be called *retro-directive reflectors*.[1]

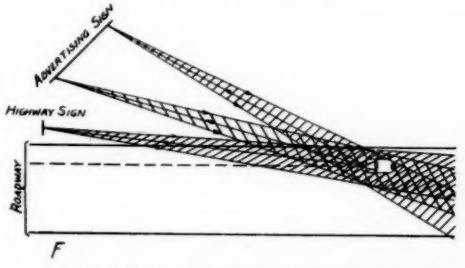

Fig. I-1. Return beams from sign reflectors.

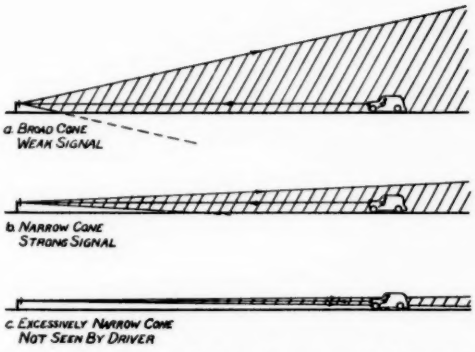

Fig. I-2. Signal strength depends on spread given return beam.

Literally, *retro-directive reflection* means exactly reversing the direction of each ray of light, sending it back along either its original path or a parallel one. An *ideal* retro-directive reflector would thus return most of the light received from a particular source back into the source itself, and it constitutes a theoretical limiting case of no practical value. The practical retro-directive reflector differs from the ideal one in giving to the return beam the conical shape required to reach the eyes of an observer; the half-vertex angle of this cone is usually of the order of one degree.

In addition to their applications in signs and as warning signals on the rear of trucks, busses, and bicycles, these reflectors have been put to many

[1] *Retro-directive* is a more appropriate term than *autocollimating*, which has been widely used in this connection, particularly in the patent literature. *Autocollimating* has an accepted meaning which is quite different from the present one [see, e.g., R. A. Houstoun, *A Treatise on Light* (Longmans, Green and Company, 1930), p. 106]. The writer coined the term *retro-directive reflector* to fill the need for an accurately descriptive term, and used it in testimony in a patent suit before the Federal District Court in Dallas, Texas, during February, 1936. The trade term *reflex reflector* is perhaps satisfactory as such, but is not sufficiently descriptive for our purposes.

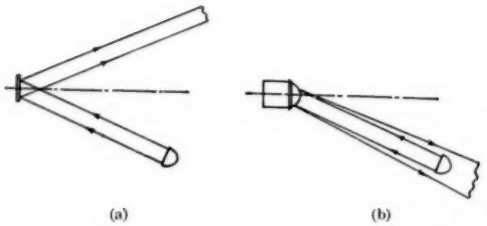

FIG. I-3. Contrasting types of reflection. (a) Ordinary reflection; (b) retro-directive reflection.

other uses. Among these are: reflecting tail light lenses, which give warning by reflection to the following driver even if the lamp within is not burning; reflector buttons built into the tail light for the same purpose; stationary warning signals at bridge abutments, dead ends, etc. Reflector buttons are also used to mark the center-line of the highway, and the recent installations for reflector delineation of highways provide reflectors on posts one hundred feet apart along each shoulder to reveal the course of the roadway for a considerable distance ahead.

Many states require retro-directive (reflex) reflectors for trucks and busses, and some of these states prohibit the sale of reflectors which do not meet certain minimum specifications. This is necessary because there are many devices on the market which are only crude imitations of retro-directive reflectors, and which perform very poorly, or not at all. Since they present much the same appearance (on the counter) as satisfactory reflectors, only such regulation can save the lay buyer from wasting his money. It is unfortunate that not all states which require reflectors have such specifications.

Emphasis has been laid in these introductory paragraphs on the functional aspects of retro-directive reflectors because that aspect is exactly the same for all types, and our main objective is the development of those broader aspects of the subject which relate to every type of practical retro-directive reflector.

B. Need for Technical Treatments

The art and science of reflector manufacture have been developed to such a degree that the effectiveness of the better reflectors, which are available from several makers, is by no means generally appreciated, even among those directly concerned with their purchase and use. In fact, very few of those who need to select the particular type of reflector best suited to a given situation, and to distinguish between adequate performance and crude imitation, are able to do so. This is largely because there is no published material from which one can gain a fundamental understanding of the rather special technical problems involved in the intelligent selection and use of retro-directive reflectors. There is serious need for a means of giving quantitative descriptions of optical properties, and for corresponding methods of measurement, which together may serve as a basis for specifications defining adequate performance in each type of service. Furthermore, there can be little doubt that extensive use of specifications in purchasing will contribute to the further development of reflector quality. The present paper is the first of a series in which the fundamental problems involved in the use, description, and measurement of retro-directive reflectors will be treated.

C. Types of Retro-Directive Reflectors

The purpose of this section is to explain briefly the manner in which retro-directive reflection is accomplished in each of the several reflector types. Since all are functionally identical, their performance will be described in the same terms and measured by the same methods, but an understanding of their principles of operation will provide helpful background and be useful in judging probable durability against weather and breakage—subjects which cannot be discussed in detail here.

1. Triple reflectors

If three plane reflecting surfaces are arranged so that each is perpendicular to the other two—like the floor and two adjacent walls of a room, meeting at a corner—any ray of light which has been reflected successively from all three surfaces will have been exactly reversed in direction, as illustrated in Fig. I-4(a). Such a *triple reflector* thus furnishes an example of a retro-directive reflector. If made precisely as described, it would be an ideal retro-directive reflector, and would not give to the return beam the conical shape which is necessary for practical applications. In case the surfaces are accurately flat, but are not precisely

perpendicular to each other, the reflected beam arising from a parallel incident beam is in general split into six separate beams, each being itself a parallel beam, but diverging somewhat from the other five sub-beams. In order to make the reflector practical, one further modification is necessary—some means of spreading each of the six sub-beams sufficiently to ensure their merging into an effectively uniform, slightly spreading, return beam. This is accomplished in several ways, for instance by placing a lens of small power, either positive or negative, in front of the reflector, or by leaving the surfaces slightly uneven.

One form of triple reflector which is used as a warning signal at danger spots on highways is a metal cup several inches in diameter. Beginning about an inch from the mouth, the sides are flattened to form the three mutually perpendicular reflecting surfaces, which meet in a blunt point at the bottom of the cup. The mouth is hermetically sealed with a concavo-convex red cover glass, and six such units mounted on a plaque at the corners of a hexagon compose the complete signal.

In another form which is more widely used, particularly for reflecting tail light lenses and for truck and bus reflectors, a mosaic of 90-degree triangular pyramids ("cube corners") is molded on the back of a piece of glass or other transparent material, as shown in Fig. I-4(b). Glass is, of course, the material most commonly used, but the slumpage and shrinkage which unavoidably follow the molding operation impose serious limitations on the precision which can be attained, and thus on the narrowness of the cone of reflected light and the effectiveness of the reflector. This difficulty has recently been met by using one of the new transparent synthetic plastics. In either case, unless the incident light is excessively oblique (makes too great an angle with a line perpendicular to the composite reflector), total internal reflection occurs at each plane surface, and each set of three surfaces acts as an independent retro-directive triple reflector. The light is refracted (bent) upon entering and also upon leaving the reflector, but if the front surface is plane, its refractive effects on the incident and emergent light compensate for each other. However, the bending of the incident beam serves to increase materially the range of incident-beam directions for which total internal reflection occurs at all three surfaces. This range of directions is sometimes further increased by coating the back surface with silver.

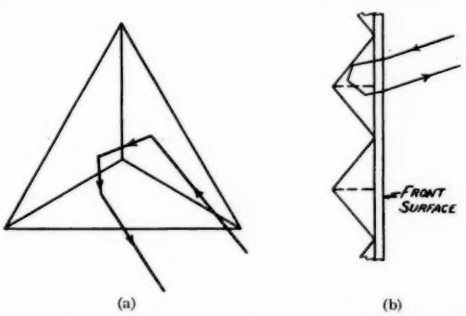

FIG. I-4. Triple reflectors. (a) Three perpendicular mirrors; (b) mosaic-type triple reflector.

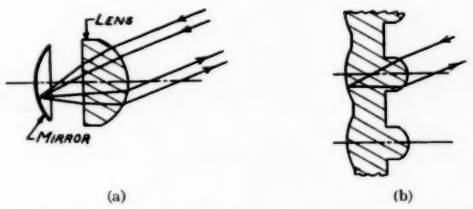

FIG. I-5. Lens-mirror button and plaque. (a) Essential parts of lens-mirror button; (b) lens-mirror plaque.

2. Lens-mirror button and plaque

The other type of retro-directive reflector which has received practical application is the lens-mirror button shown in Fig. I-5(a). Usually called simply a "reflector button," it consists of a short focus converging lens with a mirror conforming to its principal focal surface. A parallel incident beam, whether parallel to the lens axis or not, is converged to a point on the mirror and returned by it to the lens as a beam diverging from the point at which reflection occurred. Since this point is in the principal focal surface, the lens renders the emergent beam parallel, with its direction of travel just the reverse of the incident direction. This arrangement duplicates the optical system of the eye, the mirror surface corresponding to the retina. It is by means of retro-directive reflection that the eyes of a cat or other animal shine at one who is close to a light source. A reflector button is thus merely an artificial "cat's eye."

If the lens of the reflector button actually functioned in the ideal manner described in the preceding paragraph, the lens-mirror button would be an ideal retro-directive reflector without practical value. Actually, however, the defects inherent in simple spherical lenses are more than sufficient to provide that spreading of the return light which is necessary for practical applications. Mechanical considerations require the use of short focus lenses, and if such a lens is made merely with two spherical surfaces, spherical aberration makes the spread rather large. One manufacturer uses a lens having a front surface made up of several spherical zones; all zones have the same average focal length, and the amount of spherical aberration within each zone is much less than it would be if the front face were a simple spherical surface. If spherical aberration were removed entirely, however, the effects of chromatic aberration would become serious, so that the lens would show different colors to observers in different parts of the return beam; the two component achromatic lenses which are standard in the more usual optical instruments are out of the question here because of cost. Another manufacturer employs a spherical-aberration-free aspherically curved surface which is modified by toroidal rings or small random irregularities to give sufficient spread to the return beam and "smother" chromatic aberration. The location of the mirror also affects the beam spread, which is increased by moving the mirror either forward or backward from the principal focal surface.

The mirror is in some cases a polished cup behind the lens; in other cases it is a reflecting coating applied directly to the rear surface of the lens itself, in which event all the refraction occurs at the front surface. In either case, the curvature of the mirror surface has little or nothing to do with the focusing action; this surface is curved so that it may conform to the principal focal surface and may also return to the refracting surface or surfaces light which has entered the lens obliquely. The size of this surface in relation to the focal length of the lens is the feature which determines the maximum useful angle which may obtain between the lens axis and the direction of the incident light; this angle is ordinarily about thirty degrees, which is probably more than ample for highway applications. In the modification of the lens-mirror button represented in Fig. I-5(b), a number of lenses are molded side by side on one surface of a single piece of glass, each lens surface being opposed on the back, which is silvered, by the proper shape of surface to accomplish the reflection. Such a reflector may well be called a *lens-mirror plaque.*

D. Inherent Characteristics

Since it is essential to the effective use of retro-directive reflectors that their inherent characteristics be recognized and properly taken into account, it is worth while to examine more closely the principal features of their performance. When a retro-directive reflector is furnished with light from a distant source and is viewed from a point near that source, it acts as a secondary source of light; as such it has a certain effective candlepower[2] which depends principally upon three features of the situation:

(a) The illumination to which the reflector is subjected by the light source.
(b) The degree to which the light falls obliquely on the reflector, that is, the angle between the axis of the reflector and the direction of the incident light.
(c) How far the observer's eyes are above, below, or to one side of the primary light source, as measured by the angle between the line of sight and the line from light source to reflector.

1. Illumination

Other things being equal, the effective candlepower of a reflector is directly proportional to the illumination (footcandles) being produced at the location of the reflector by the particular source being employed, and so it is highly advantageous to locate the reflector where it will receive as much light as possible. Since the illumination produced by well-adjusted head lights diminishes very rapidly with increasing distance above the roadway, reflecting signs, truck and bus reflectors, etc., are most effective when placed as low as other considerations permit. A limit is set in this direction, however, by the effect of dirt on

[2] A critical discussion of the applicability of this concept to such a situation will be given in Part II of this series of papers.

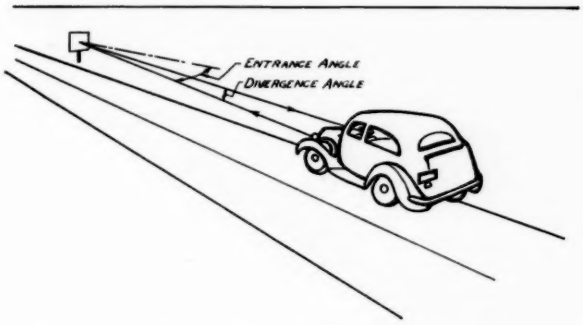

FIG. I-6. Entrance and divergence angles.

the face of the reflector, for if placed close to the roadway it will receive an inordinate amount of dirty spray in wet weather. Dirt on the face of a reflector is doubly effective in reducing its efficiency, for the light must pass through the film of dirt twice—once on the way in, and again on the return trip—so it is even more important for reflectors to be kept clean than for head and tail lamps. Even a moderate film of dirt will rob a reflector of ninety percent of its effectiveness, a fact which is important in determining the requirements for reflectors to fill a particular need.

2. Obliquity, or entrance angle

The ability to reflect obliquely-incident light back into the neighborhood of the source is the characteristic which distinguishes a retro-directive reflector from all others, but there is in nearly all cases some falling off in effective candlepower with increase in obliquity. This effect is usually not serious for *entrance angles* (see Fig. I-6) of 10 degrees or less, and most commercial reflectors are effective at entrance angles up to 25 or 30 degrees—a range considerably exceeding that ordinarily needed. The sign or signal is most effective when arranged so that it is illuminated and viewed within 10 degrees of the reflector axis for as much of the time as possible, but it is best never to let the light strike a sign exactly at right angles to its plane (zero entrance angle), for glare reflection from the sign face would then seriously interfere with the visibility of the reflectors. This difficulty is avoided for signs beside the road when they are arranged at right angles to the roadway, as is customary. If a sign is located near a curve and on the continuation of a line of travel, it should be set at an angle of two or three degrees to that line.

3. Location of observer with respect to source; divergence angle

It has been assumed so far that the intensity is constant within the narrow cone of light—the "searchlight beam"—which the reflector automatically aims back at the source, or in other words, that the reflector would appear to have the same candlepower to all observers whose eyes are within this cone. Actually, this is not at all the case, for in almost all designs the return beam is strongest at points on its axis, and is weaker and weaker at points farther and farther removed from its axis. A reflector will thus show different values of candlepower to observers viewing it from different positions within the return beam, appearing brightest to the one whose line of sight makes the smallest angle with the direction from which light arrives at the reflector. An observer for whom this *divergence angle* (see Fig. I-6) is much more than one or two degrees will receive little or no light. For this reason, retro-directive reflectors are suitable for use in those situations, and those only, where the observer's eyes are so close to a light source that the reflector is viewed at a smaller divergence angle—the smaller the divergence angle, the better the results. The head lamps of an automobile are so placed that their average distance from the driver's line of sight is usually about thirty or forty inches (average for the two lamps); the corresponding divergence angle is two degrees when the automobile is 75 or 100 feet from the reflector, one degree at 150 or 200 feet, and becomes proportionately smaller at larger highway distances. The small angle requirement is thus automatically satisfied for the driver of an automobile, and usually also for the other occupants, at all highway distances above a

reasonable minimum, essentially because observer and light source move along together. This point must be kept clearly in mind when considering other possible applications for retro-directive reflectors.

As an automobile approaches a reflector from a great distance, moves closer, and finally approaches to within 100 feet, the effective candlepower of the reflector will vary quite widely, even though the two effects which have just been discussed tend to compensate for each other. At distances of several thousand feet, the divergence angle is practically zero, so that the reflector is being viewed in the most advantageous manner, but the illumination furnished to the reflector is so small that the reflector may be invisible, or nearly so. As the distance becomes smaller the illumination increases, at first slowly and then more rapidly, while the divergence angle remains so small that the reflector is practically as effective, per unit of illumination, as before; thus the effective candlepower increases with the illumination, and the reflector becomes clearly visible. In the middle range of distances—say in the hundreds of feet—the illumination furnished by the head lamps increases quite rapidly, but now the divergence angle has grown until the driver's eyes have left the strongest part of the return beam, and are moving into weaker and weaker regions. To the extent that the increase of illumination balances the effect of increase in divergence angle, the effective candlepower of the signal or sign will remain constant. Upon still closer approach, however, the quite rapid increase of illumination is more than counterbalanced by the weakening effect of the growing divergence angle, for the driver's eyes pass to the outer fringe of the return beam, and finally move outside it. The effective candlepower value under given conditions, and also the location and extent of the middle range of distances—the range within which the variations of illumination and divergence angle are such as substantially to balance each other—depend enormously upon the detailed design of the reflector and the precision with which it has been manufactured. For instance, reflectors which are to function effectively between a few hundred feet and one or two thousand feet must confine practically all the light in the return beam within an angle of half a degree or less, a result which requires the utmost in manufacturing precision.

E. The Quantitative Description of Reflector Performance

In the last section, we discussed the three principal factors which determine the effectiveness of a given reflector in a particular situation—illumination, entrance angle, and divergence angle. We also described the manner in which this effectiveness depends upon each factor, insofar as the dependence is the same for most commercial retro-directive reflectors. Let us now consider the quantitative description of individual reflectors, to the end that one type or brand may be compared with another.

1. Candlepower per footcandle, or specific intensity

Since the effective candlepower of any reflector is proportional to the illumination to which it is subjected, its performance is appropriately measured by the candlepower of the reflector *per footcandle of illumination upon it*. A suitable name for the candlepower per footcandle value is *specific intensity*, for it is obtained by dividing the luminous intensity of the reflector (considered as a secondary source of light) by the illumination on the reflector. For any given situation, the effective candlepower to be expected of a particular reflector will then be the product of its specific intensity by the illumination (footcandles) to be furnished by the head lamps. It will be clear from what has already been said that different specific intensity values must be used for situations involving different entrance angles or different divergence angles. The complete description of a reflector would thus include a table of values or a set of curves giving the specific intensity for every entrance angle and every divergence angle.

2. Long range specific intensity

For most purposes, at least, the most significant single number which can be used to give a partial description of the optical performance of a retro-directive reflector is its specific intensity (candlepower per footcandle) at zero entrance angle and at very small (virtually zero) divergence angle. This will correspond to a highway

situation in which the distance of the reflector from source and observer is 500 feet or more, and may well be called the *long range specific intensity*. It is usually, but not always, higher than the specific intensity at any other combination of entrance and divergence angles.

3. Entrance-angle characteristic

We have already seen that the ability of a reflector to function retro-directively at various entrance angles is the essential characteristic which makes a reflector useful for the purposes considered here. One may compare different reflectors in this respect by measuring the specific intensity of each at various entrance angles, say every 5 degrees out to 25 or 30 degrees. These measurements need to be made with the reflector axis both to right and left of the direction of the incident light, and also above and below, but the measurements above and below need not extend past 10 degrees. Such a set of measurements will be referred to collectively as an *entrance-angle characteristic*. It is convenient and suitable that these measurements be made at some one divergence angle (Fig. I-6), which may appropriately be chosen near the middle of the range of divergence angles which will be employed in service. It is better not to use an extremely small divergence angle because reflectors are rarely viewed obliquely from great distances.

Although it is of first importance that a reflector should have a satisfactory entrance-angle characteristic, this property usually does not provide a decisive basis for choosing one reflector over another, for most of the reflectors widely sold are about equally satisfactory in this respect. The specific intensity may fall at 25 or 30 degrees to one-half or one-fourth its zero-entrance-angle value, but even when this ratio differs appreciably from one reflector to another, its differences are usually far less important than differences in long-range specific intensity and in the characteristic described below.

4. Divergence-angle characteristic

Of the very greatest importance is an account, more or less detailed, of the manner in which the specific intensity depends upon divergence angle at zero entrance angle. Such an account will be called a *divergence-angle characteristic*. Since the illumination at a reflector is inversely proportional to the square of its distance from the source, great service distances entail small illuminations. But great service distances also correspond to small divergence angles, at which the specific intensity of the reflector is high. Thus the variation of specific intensity with divergence angle tends to offset the corresponding variation of illumination with distance—an effect previously described in detail—but the degree to which these two factors actually do compensate for each other varies enormously from one reflector to another. In other words, there are wide variations among the divergence-angle characteristics of different commercial reflectors, so that knowledge of these characteristics is of first importance for comparing reflectors with each other.

A divergence-angle characteristic is conveniently represented by a curve with a *logarithmic scale* for specific intensities. The logarithmic scale is a practical necessity because of the wide variations which occur (and which for many uses *should* occur) in the specific intensity values from one divergence-angle to another. When a uniform scale is used, the low values crowd so close to the zero line that significant differences between values for different reflectors may appear to be negligible, as is illustrated in Fig. I-7(a). On a logarithmic scale, however, a given percentage difference is represented by the same *distance*, at whatever part of the scale it may appear, so that the auxiliary scale in Fig. I-7(b) can be transferred to a card and applied vertically to any region of that graph to read off the ratio of two values. It has been found convenient to use 3-cycle semilogarithmic graph paper for plots of both divergence- and entrance-angle characteristics.

5. Test methods

Since a survey of the methods which may be used for obtaining the quantitative information outlined above will be given in the third paper of this series, only brief mention will be made here of the main features of the problem. In the first place, it should be clearly recognized that measurement of the specific intensity of a retro-directive reflector is a task requiring the resources and techniques of a well-equipped photometric laboratory, and should not be attempted as an off-

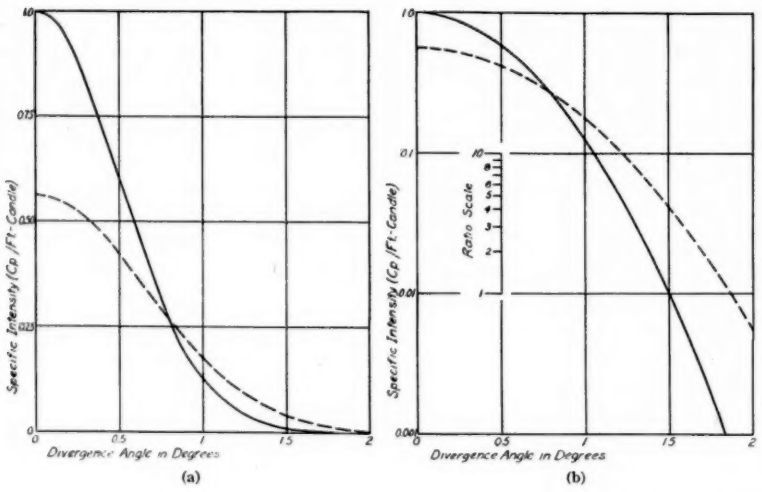

FIG. I-7. Divergence-angle characteristics. (a) Uniform vertical scale; (b) logarithmic vertical scale.

hand and occasional procedure. Furthermore, this particular type of photometric measurement requires the development of special apparatus and methods, even in a professional laboratory. It will be clear, then, that such measurements are not to be undertaken lightly.

In spite of these difficulties, very revealing qualitative comparisons between reflectors can readily be made with simple equipment, provided that proper care is exercised and the inherent characteristics of retro-directive reflectors are kept clearly in mind. Two or more reflectors can be set up close together and 25 feet or more from the observer, either in the dark or in a dimly-lighted room, and illuminated by a flashlight producing a fairly uniform spot of light. If one then sights down the barrel of the flashlight, putting the base of the flashlight just in front of his nose, he will be viewing the reflectors at a divergence angle of about one-half degree (an angle of one degree corresponds to a separation of $5\frac{1}{4}$ inches at 25 feet). As the flashlight is gradually lowered, observations may be made at increasing divergence angle, and the reflectors will usually be seen to become less and less bright. If two reflectors with very different divergence-angle characteristics are compared in this way, the one which shows the larger effective candlepower at smaller divergence angles will usually show the smaller effective candlepower at large divergence angles. By walking first to one side and then to the other, one can easily compare the reflectors with respect to their entrance-angle characteristics. It is best to make such comparisons only between reflectors of the same size, for the eye cannot compare, or even recognize equality between, the amounts of light coming from areas of different sizes, unless both are sufficiently far away to appear as points. When it becomes necessary to make such comparisons, one should work at a much greater distance or use a reducing glass. Flashlight tests may also be misleading with regard to residual color effects in clear (crystal) reflectors. If present at all, such colors will be much more prominent here than under actual road conditions; this point will be treated more fully in Part II.

Attempts to gain a rough idea of the quality of a reflector by standing with one's back to the light and looking into the reflector, held at arm's length, are most unfortunate, for the distance separating the pupil of the eye from the side of the head is such that no reflected light can reach the eye at divergence angles less than several degrees. A good reflector will therefore appear quite dark when so viewed, although a very poor one, which sends back most of the light at divergence angles far too large for effective use, may appear fairly bright.

Several attempts have been made to devise methods of testing reflector buttons by nonvisual methods which utilize photoelectric cells and electric meters. Because of the small amounts of light available and the limited sensitivity of

suitable photoelectric cells, it has been necessary in even the best[3] of these methods to collect all the light flux returned at useful divergence angles and to measure simply its total amount. In certain other methods, there is no effective control whatever of the divergence angles at which light is accepted. In either case, the results consist of meter readings obtained at various entrance angles. In the best case, the meter reading can indicate only a sort of average effectiveness for the whole useful range of divergence angles, while in the other cases the button which gives the largest reading is not unlikely to perform worst under service conditions. With none of these methods is it possible to obtain the divergence-angle characteristic—the characteristic which has been recognized as a most important basis for comparing different reflectors. A more detailed analysis of these methods will be given in Part IV of this series, where it will be shown that any such method is quite unsuitable for use as the sole, or even as the principal, basis of specifications. However, such methods offer promise for making before-and-after tests on reflectors subjected to accelerated treatments designed to test the permanence of their optical properties, and perhaps for rapid checks on the uniformity among the various reflectors of a given set of samples. In the latter case, however, it will be necessary to use extreme care in interpreting the readings. It is unfortunate, but true, that properly interpretable tests of retro-directive reflectors are quite difficult to make.

6. Interpretation of test results

There is a very real danger to be guarded against in the interpretation of test results—the danger of giving an unwarranted significance to differences which may appear to be substantial, but which are actually negligible from the practical standpoint. It is difficult, to say the least, for the ordinary person to choose the stronger one of two light sources when they are side by side and differ in candlepower by twenty-five percent; it is quite impossible if he looks first at one and then at the other after an interval of a few seconds. Obviously, then, values which differ by amounts much less than twenty-five percent are practically identical, and for many purposes the difference of two values should be at least fifty percent of the lower one before this difference is to be considered as serious. However, these remarks must not be taken to apply to the meter readings discussed in the preceding paragraph of this section; the situation in such cases may be entirely different, but discussion of the reasons for this possible difference must be postponed.

[3] R. Kingslake, J. Opt. Soc. Am. **28**, 323 (1938).

F. Specifications

It is certain that no single number can suffice adequately to describe the performance of a given reflector, but it is also clear that the amount of data required to give a complete description is so large as to be prohibitive for specification purposes. A specification which is neither intolerably burdensome nor totally inadequate must represent a compromise. If minimum values are to be specified, careful attention must be given to the results desired, to the products which are available, and to the products which can be made available, if and when they are demanded.

Some progress has already been made. Specifications and a test method for reflectors to be used as warning signals on motor vehicles were developed by a joint committee of the Society of Automotive Engineers and the Illuminating Engineering Society, and were adopted by the Societies in 1931.[4] These specifications, having served as models for the regulations adopted by a number of state commissions, have performed a very valuable service, and their revision is now under consideration. Specifications and test methods for reflector buttons have been developed by several state highway departments, but not all this work has been sound, and it seems to be generally agreed that a serious need exists for major improvements in this situation.

The present paper has attempted only the preliminary exploration necessary to the development of a general basis for optical specifications and tests. This general basis will be completed and critically discussed in the next paper, which will be followed in Part III by a survey of the test methods available for measuring specific intensity and in Part IV by a study of flux-measuring test methods.

[4] *S. A. E. Handbook* (Society of Automotive Engineers, 29 West 39th Street, New York, 1939), p. 611.

II. General Basis for Optical Descriptions, Specifications, and Tests

The basis, begun in Part I, for giving quantitative descriptions of reflector performance in terms of specific intensity and its dependence upon the manner in which the reflector is illuminated and viewed is completed, the definitions of all special terms being first tabulated and then discussed. Inquiry into the validity of specific intensity as a measure of reflector performance shows that this validity depends upon the applicability of the inverse-square law to the reflected beam at distances encountered in service; available information allows the working hypothesis that the inverse-square law applies at distances of 100 feet and greater. The suitability of specific intensity is also pointed out, together with the special considerations entailed by the presence of a *pair* of light sources in the typical highway situation. It is shown that the specific intensity at zero divergence angle is inversely proportional to the angle of spread which the reflector gives to the return beam.

AN introductory treatment of the reflector buttons, triple reflectors, etc., which are used in various ways on highways and vehicles was given in the first paper of this series,[1] where a number of terms needed for describing the optical properties of such *retro-directive* reflectors were proposed. The principal feature of the description, the specific intensity (candlepower under one footcandle illumination), was seen to depend upon the particular manner in which the reflector is illuminated and viewed. Only those terms necessary for describing the most important aspects of the illumination-viewing situation were introduced, and the purpose of the present paper is to complete the general basis for an optical description around which suitable specifications and tests can be built.

A. Complete Description of Source-Reflector-Observer Situation, and of Reflector Performance

The following table gives proposed names, definitions, and symbols for a set of terms sufficient for a complete description of both the source-reflector-observer arrangement and the optical properties of the reflector, either under service conditions or in the testing laboratory. The set of geometrical terms has the following desirable features:

(a) Completeness—laboratory results will be reproducible, and can be made to correspond to service conditions in every significant detail, except, of course, the motion of the car along the highway, which creates a changing situation rather than a static one.

(b) Adaptability—all service situations and laboratory arrangements can be described.

(c) Simplicity—laboratory arrangements already in use are described simply and naturally. Most of the terms will be discussed at the end of the table; those introduced and discussed in Part I are marked with an asterisk.

1. Definitions of terms

("Light source" and "reflector" refer to their respective centers.)

Terms describing the geometry of the arrangement of source, reflector, and observer's eyes. Illustrated in Fig. II-1.

Symbol	*Term*	*Definition*
S	Highway distance	Distance, in feet, from light source to reflector in a particular service situation.
S	Test distance	Distance, in feet, from light source to reflector undergoing test in laboratory.
γ	*Entrance angle	Angle at reflector between direction of light incident on it and direction of reflector axis.
δ	*Divergence angle	Angle at reflector between observer's line of sight and direction of light incident on reflector (strictly, from a single point in the source).
α	Azimuth angle	Angle, measured counterclockwise as viewed from source, from plane containing eye, light source and reflector to plane containing reflector axis and light source.
β	Orientation angle	Angle fixing orientation of reflector with respect to rotation about its own axis, measured in manner described under "Discussion of Definitions."

[1] "I. Fundamental Optical Properties," to be referred to as Part I.

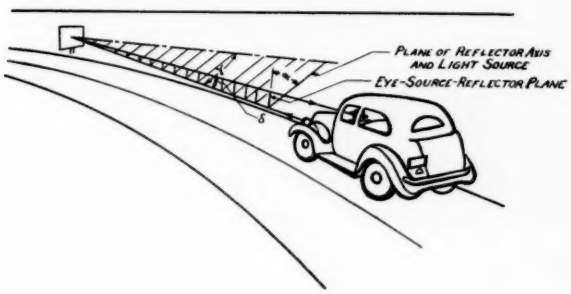

FIG. II-1. Entrance, divergence, and azimuth angles.

Terms describing the irradiation of the reflector.

Symbol	*Term*	*Definition*
C	Source candlepower	The beam candlepower of the light source, in the direction of the reflector.
E	*Illumination	Illumination, in footcandles, produced at the reflector by the light source. $E = C/S^2$

Terms describing the response and performance of the reflector.

Symbol	*Term*	*Definition*
C'	*Effective candlepower	Candlepower of the reflector, considered as a secondary light source, when under a particular illumination, and when illuminated and viewed from particular directions.
I	*Specific intensity	Effective candlepower divided by illumination at reflector. Measured in candlepower per footcandle. $I = C'/E$.
I_0	*Long range specific intensity	Specific intensity at zero entrance angle and small (practically zero) divergence angle. Measures the performance of the reflector under most service conditions involving large highway distances.
	*Entrance angle characteristic	A curve showing the specific intensity of a particular reflector plotted against entrance angle. Taken at specified divergence, azimuth, and (if appropriate) orientation angles.
	*Divergence angle characteristic	A curve showing the specific intensity of a particular reflector plotted against divergence angle. Taken at zero entrance angle unless otherwise specified, and at specified azimuth and (if appropriate) orientation angles.
	Angular range, or entrance limit	Value of entrance angle at which specific intensity falls to 10 percent of its zero-entrance-angle value. Other conditions as for entrance-angle characteristic.
D	Divergence limit	Value of divergence angle at which specific intensity falls to 1 percent of its value at small (practically zero) divergence angle. Other conditions as for divergence-angle characteristic.
	Residual color	In clear (crystal) reflectors, difference between color of source and color manifested by reflector under conditions of service—see last paragraph of Section C.

2. Discussion of definitions

**Entrance, *divergence, and azimuth angles.*—All three of these angles (or some other equivalent set) are necessary for a complete determination of the location and aiming of the reflector with respect to light source and observer. Since retrodirective reflectors are not aimed in service, the effects of changing the relationships of source, observer, and reflector axis must be considered. The divergence angle locates the observer with respect to the source, while the entrance angle measures the obliquity of the reflector axis to the direction of the incident light, and the azimuth angle completes the location of the reflector axis, taking on different values if that axis is rotated about the source-reflector line. If the observation point is directly above the light source (as is usually the case in the laboratory) then azimuth angles of 0°, 90°, 180°, and 270° will correspond to rotating the reflector axis so that it is above, to the left, below, and to the right, respectively,

of the light source, as seen by a person who is facing the reflector from behind the source. For a driver whose line of sight passes directly above a single head lamp, azimuth angles of 0° and 180° will correspond to his direct approach to a reflector while he is ascending and descending a hill (reflector axis horizontal), and azimuth angles of 90° and 270° will correspond to reflectors on the left and right sides, respectively, of a level road. The azimuth angle is of secondary importance at small entrance angles, but requires consideration at larger values because of the astigmatism of a lens-mirror unit at oblique incidence.

Orientation angle.—This angle locates the reflector with respect to rotation about its own axis, and is thus unnecessary for a reflector button with perfect circular symmetry, but is of some importance for buttons with slightly malformed lenses and for triple reflectors (which do not have circular symmetry) in order that laboratory results may be accurately reproduced and correlated with service conditions.

Definition.—The orientation angle lies in the plane of the reflector aperture and is referred to the line in which this plane intersects the eye-source-reflector plane, being measured counterclockwise (as viewed from the source) to a line fixed in the reflector and intersecting the reflector axis. The latter line passes through the projection of the "TOP" mark if there is one on the reflector, otherwise it is an arbitrary line.

The meaning of this definition is illustrated in Fig. II-2, which represents a laboratory reflector mount designed on the supposition that the eye-source-reflector plane is vertical and perpendicular to $B-B'$. Since rotations about $A-A'$ and $B-B'$ leave $A-A'$ in this plane, entrance and azimuth angles can be changed by such rotations without affecting the orientation angle, which is altered only by rotation of the reflector within the ring. Such would not be the case if the stem of the yoke were vertical. It may be remarked that the orientation angle has been chosen with an eye to simplicity of description of both service and laboratory situations. If the reflector has its axis horizontal in service, with its "TOP" mark up, and if the eye-source-reflector plane is vertical, the orientation angle will be zero for every manner of approach to the reflector. For the

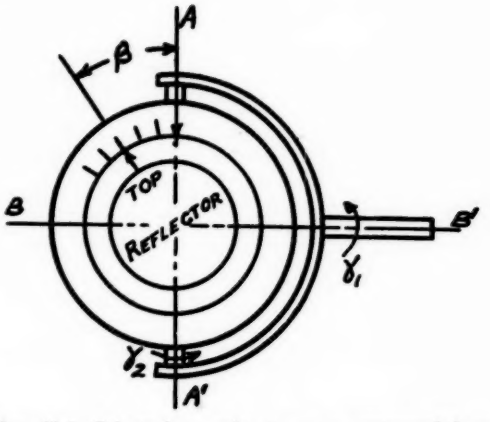

FIG. II-2. Orientation angle, β. γ_1, γ_2 measured from normal position shown.

reflector mount of Fig. II-2, when $\alpha=0$, $\pm 90°$, or 180°, one of the angles γ_1, γ_2 is zero and the other equals $\pm\gamma$ as becomes obvious upon consideration. In general,

$$\sin \gamma_2 = -\sin \alpha \sin \gamma,$$

and

$$\cos \gamma_1 = \cos \gamma/(1-\sin^2 \alpha \sin^2 \gamma)^{\frac{1}{2}}.$$

Source candlepower—enters the computation of the illumination to be expected at the reflector in a given laboratory or service situation. The variation of head lamp candlepower with direction must of course be considered.

**Illumination*—to be computed without regard to entrance angle. The effect of obliquity on the amount of light received is included in the variation of specific intensity with entrance angle.

**Effective candlepower*—measures the signal value of the reflector under service conditions, and is the candlepower which is to be measured in the laboratory. In either case, it depends upon illumination and upon the entrance, divergence, azimuth, and (perhaps) orientation angles, and of course, varies from one reflector to another when all these quantities are kept constant.[2]

**Specific intensity*—measures the performance of the reflector, giving its response to the reference illumination of one footcandle. In a given situation, the effective candlepower to be expected of a reflector with known specific intensity

[2] See also Section B.

will be obtained by multiplying its specific intensity by the illumination to which it will be subjected. The specific intensity depends upon the various angles just as the effective candlepower does, but not upon the illumination.

**Long range specific intensity.*—More nearly than any other single number, this quantity gives a concise measure of the general effectiveness of a reflector. However, it alone cannot be relied upon, for there must be at least some consideration given to the next two properties discussed.

**Entrance-angle characteristic.*—The deviation of this curve from a horizontal straight line measures the reflector's loss of effectiveness when it is illuminated and viewed obliquely. This effect is nearly always substantial, but is not too serious in a good reflector. Since the entrance-angle characteristic will, in general, change somewhat as the azimuth angle is changed, it is sometimes well to obtain such information at (say) four azimuth angles.

**Divergence-angle characteristic.*—This curve usually shows a more-or-less rapid increase of specific intensity with decreasing divergence angle. The combined effect of this variation and the accompanying variation of illumination with highway distance (small divergence angles correspond to large highway distances, and vice versa) was discussed in detail in Part I. Just how rapid an increase is desirable depends upon the particular type of service for which the reflector is intended.

Angular range, or entrance limit—a gross feature of the entrance angle characteristic which measures the range of entrance angle throughout which the reflector is effective. It may either be read off the entrance angle characteristic or be determined more directly, and will in general depend upon azimuth angle and perhaps slightly upon divergence and orientation angles as well.

Divergence limit—a gross feature of the divergence-angle characteristic which furnishes a nominal measure of the range of divergence angle throughout which the reflector is effective, or of the rapidity with which the specific intensity increases with decreasing divergence angle (the smaller the divergence limit, the more rapid the increase).

Although it is customary in the photometry of projectors to consider the edge of the beam to be located where the intensity falls to 10 percent of the maximum value, such a convention would not be appropriate for retro-directive reflectors because of the large range of illuminations to which a given reflector is subjected at various service distances. Compare, for instance, situations at 600 feet and at 75 feet. The reflector will receive at 75 feet an illumination 64 times as great as at 600 feet, and this increase in illumination will compensate for a decrease in specific intensity to $\frac{1}{64}$, or 1.5 percent of the long range value. Therefore, unless the specific intensity at the larger divergence angle (smaller distance) is less than 1.5 percent of its maximum value, the effective candlepower at the smaller distance will be at least as great as at the larger distance, in spite of the small specific intensity. Thus a specific intensity which is too small to be of value at one service distance is quite adequate at another. This is the reason for basing the definition of divergence limit on so small a value as 1 percent.

It should be noted that if one reflector has a much higher long range specific intensity than a second, the first will have as high a specific intensity outside its divergence limit as does the second at its divergence limit. Until an absolute minimum for useful specific intensity is found, such difficulties can hardly be avoided.

While it is usually desirable to have as large values as posisble for both divergence limit and long range specific intensity, it will be shown later in this paper that either can be increased only at the expense of the other. Since no single reflector can have large values for both, they must be considered together—never either alone.

The foregoing scheme for describing reflector performance may perhaps seem more elaborate than is fitting or necessary. The attempt has been to provide, not only terms suitable for specifications, but a convenient, unambiguous, and reasonably complete language for discussion of fundamental questions, such as those which must be considered during the formulation of specifications pertaining to any particular class of service. It may well be that less specialized language than, for instance, "azimuth angle," will suffice in the specifications themselves, especially if only one or two values of the quan-

tity concerned occur, but the formulation of specifications will be facilitated by a general means of correlating service and laboratory conditions.

B. Specific Intensity, Effective Candlepower, and Inverse-Square Law

It has been tacitly assumed so far that the use of specific intensity as a measure of reflector performance is justifiable, but a fundamental inquiry into this question can be postponed no longer. Since specific intensity (effective candlepower divided by illumination) will be independent of the particular illumination used only if the effective candlepower is proportional to the illumination, we are thus faced with two questions:

(a) Is it legitimate to describe a particular reflector, when subjected to a particular illumination from a given direction, as a secondary light source with a definite candlepower in each direction from which it may be observed, regardless of the distance at which the observation is made? If so,

(b) Is this candlepower in a given direction proportional to the illumination on the reflector?

Since the second question is much the simpler one to answer, let us consider it now, presuming for the moment an affirmative answer to the first. Suppose that we make two tests of one reflector, illuminating it each time by a parallel beam of light from the same direction, changing only the amount of illumination. Regardless of the type of the retro-directive reflector which is being tested, the light will be subjected to refraction, reflection, and absorption. In each of these processes the intensity of the useful beam (refracted, reflected, or transmitted beam, respectively) is strictly proportional to the intensity of the beam giving rise to it, so that the intensity of the beam which finally emerges is proportional to that of the original incident beam. Consequently the result of any sort of photometric measurement on the emergent light will be proportional to the incident illumination. The assumption of a parallel illuminating beam was made only to simplify the description; the argument applies just as well to any pair of measurements involving irradiation from the same *range of directions*, if the amount of illumination at the reflector is the only condition changed.

Returning to the first question, we must consider the basic meaning of the term *candlepower* and its connection with the *inverse-square* law. This law applies to a point source which radiates uniformly in all directions, and states that the illumination E (footcandles) produced by such a source of candlepower C (candles) on a surface at right angles to the rays is inversely proportional to the square of the distance x (feet) between source and surface, the illumination being given by the expression $E = C/x^2$ and being independent of direction. Conversely, the product of illumination by distance squared gives the candlepower ($C = Ex^2$)—the same value for all distances and all directions. For an extended source of irregular shape, it is found that the product of illumination by distance squared is again a constant *at sufficiently great distances* (say at least 20 times the greatest dimension of the source) *and in a fixed direction*, but this product has different values for different directions. In other words, the inverse-square law holds at sufficiently great distances from an extended source, but it assigns to such a source a candlepower which depends upon the direction from which the source is viewed. For points relatively close to an extended source, Ex^2 depends upon the distance, and must be called the *apparent candlepower* corresponding to that particular distance and direction. The concept of candlepower thus possesses its full validity and usefulness only in circumstances to which the inverse-square law is applicable, and by question (a) we ask, in effect, whether the return beam from a commercial retro-directive reflector obeys the inverse-square law at distances within the service range—say 100 feet and larger.

The usual photometric rule that the inverse-square law may safely be applied at distances more than 20 times the maximum dimension of the source (here the reflector) does not apply, for the variation of beam intensity with direction is tremendously greater than in the type of photometry where this rule obtains. Although this problem has much in common with the photometry of projection systems, the irregularities, etc., rule out calculations along these lines. Apparently our question can be answered

conclusively only by experiment, and the experimental work required for a complete investigation would be both difficult and laborious.

On the other hand, the inverse-square law will certainly apply at sufficiently great distances[3] and the writer proposes the working hypothesis that *for distances of 100 feet and greater, the reflected beam obeys the inverse-square law.* If this be granted, the use of specific intensity as a measure of reflector performance is justified, at least so far as validity is concerned. Of course, this hypothesis may require revision in the light of further experimental data, but if it ever becomes untenable, the problem of describing and measuring the optical properties of retrodirective reflectors will become even more complicated than it already is.

C. Suitability of Specific Intensity. Dual Light Source. Residual Color

There remains the question whether the specific intensity is a *suitable* measure of reflector performance—that is, whether the effective candlepower properly measures the indication given by a particular reflector under given circumstances. Since the effective candlepower is, according to our working hypothesis, the candlepower of the actual source which would present the same appearance to the observer, such a description is as objective and significant as seems to be possible. The complication that both effective candlepower and specific intensity depend upon the relative angular locations of source, observer, and reflector (and perhaps also upon the orientation angle) is inherent in the problem and cannot be avoided without fatal oversimplification. It therefore seems justifiable to consider the specific intensity as a valid and suitable measure of reflector performance.

Although the light incident on the reflector has thus far been considered as coming from a single direction, under ordinary service conditions it comes from two extended effective sources—two automobile head lamps—and this fact entails certain complications. The eyes of the driver receive light which has been returned at a variety of divergence angles, a different value of divergence angle for each element of head lamp effective area. We thus have to consider the superposition of many contributions at different divergence angles. Now a single head lamp is sufficiently small, compared to its distance from the driver's line of sight, that the variation of divergence angle as referred to different elements of its area is negligible, and we can treat each head lamp as if concentrated at its center. But the placing of the two head lamps is usually such that the divergence angle at the eye position is considerably different when referred to one than to the other. In fact, this is more or less true of the entrance, azimuth, and orientation angles also.

The strictly correct way to meet this difficulty is to compute all four angles with reference to each head lamp separately, and to multiply the specific intensity corresponding to each set of values by the illumination furnished the reflector by the head lamp concerned, adding the results to obtain the effective candlepower. Such an elaborate procedure is usually not justified, but the two values of each angle may be averaged and effective candlepower approximated by multiplying the specific intensity corresponding to the set of average angular values by the total illumination at the reflector. If still further simplification is deemed necessary, the variation in azimuth and orientation angles from one highway situation to another may be ignored. Another possible simplification is to ignore the right-hand head lamp, since to it corresponds the larger divergence angle and the smaller contribution to the effective candlepower. In this case, the illumination due to the left-hand head lamp only is to be considered. We can be as realistic as we like in giving an accurate description of service conditions, but we must pay the corresponding price in degree of elaboration. Closely related questions must be considered in connection with laboratory testing, and they will be taken up in Part III of this series.

There is one other respect in which it is important to recognize that service conditions involve a pair of head lamps, rather than a single one. Some clear (crystal) reflector buttons appear

[3] The dimension which is significant in determining what distances *are* "sufficiently great" in a case involving a given divergence limit is the aperture diameter of the reflector. In the case of a mosaic-type triple reflector or a lens-mirror plaque, this dimension will be that of a single sub-unit, since the different sub-units function quite independently of each other.

to be slightly colored when illuminated from a single direction; this results from incomplete "smothering" of chromatic aberration in the lens, and the color manifested varies with the divergence angle at which the button is observed. Consequently, when two substantially different divergence angles are employed simultaneously, as when two head lamps provide the illumination, there is a superposition of colors which tends strongly to suppress any deviation from white. Laboratory observations made with a single light source are thus not to be trusted in regard to residual color, and a test for color absence must be made either with a pair of sources properly arranged in the laboratory or with actual head lamps on the road.

D. Long Range Specific Intensity *Versus* Divergence Limit

If two reflectors have the same area, and thus receive the same amount of light flux under a given illumination, the one which has the larger divergence limit will spread that light flux more widely than the other; it will thus produce a correspondingly smaller illumination at the observer's eye or measuring instrument, and by both appearance and measurement will have the smaller effective candlepower and therefore the smaller specific intensity.

In order to obtain quantitative relations, we shall first treat two special cases which are of some interest in themselves, and then give the form of the relation which applies in the general case, placing the mathematical details in an appendix at the end of the paper. Suppose first that the return light is distributed uniformly throughout a narrow cone with vertex at the reflector (which is to be thought of on the large scale as practically a point) and with half-vertex angle D' degrees—the divergence limit.[4] Such a reflector would have the same specific intensity at all divergence angles less than D', where the specific intensity would drop to zero, and the specific intensity would be independent of azimuth and orientation angles. The entrance angle is to be considered constant throughout this discussion. The calculation given in Appendix I shows that in this case the specific intensity would be given by the expression:

$$I = 7.3kA/D'^2, \tag{1}$$

where A is the aperture area in square inches, and k is the fraction of the total amount of light flux received which is not lost by absorption or extraneous reflection; k may well be called the *efficiency* of the reflector. Since the divergence limit, D', appears to the second power in the denominator, *the specific intensity is inversely proportional to the square of the divergence limit*, at least in the present case of a specific intensity which is constant within the divergence limit.

For the second special case we assume that the specific intensity is maximum (I_0) at zero divergence angle (long range specific intensity) and falls off at larger and larger divergence angles; this is in general accord with the facts in typical cases. Of course, the specific intensity falls to 1 percent of its maximum value at the divergence limit, by definition. In order to proceed with the treatment, however, we must make a more definite assumption about the divergence-angle characteristic, and the simplest assumption which will satisfy the requirements is that it has the shape of the well-known Gauss error function.[5] It is shown in Appendix II that under this assumption,

$$I_0 = 33.4kA/D'^2, \tag{2}$$

where the symbols on the right side of the equation have the same meanings as in Eq. (1). Here again we have an inverse proportionality to the square of the divergence limit, but now it is the long range specific intensity which enters the relation. The only other difference between Eqs. (1) and (2) is in the value of the numerical constant by which the rest of the right-hand side is to be multiplied; this, and this only, will change when we make different specific assumptions

[4] In all cases where it is necessary that an angular value be expressed in degrees, the symbol will be accented. Where unaccented symbols for angular quantities appear, any units may be used unless a particular unit is explicitly mentioned.

[5] There is no theoretical reason for preferring this particular shape over any other which satisfies the general requirements, but this one is mathematically convenient. Fig. I-7 (Part I) illustrates the assumed shape, both to a uniform and to a logarithmic vertical scale. The curves shown by full lines in that figure correspond to $D'=1.5°$ and those shown by broken lines to $D'=2°$; the relative peak values (I_0) are adjusted in accordance with Eq. (2). The corresponding mathematical expression is given in Section 2 of the mathematical appendix which follows.

about the detailed shape of the divergence-angle characteristic.

The general relationship, which is suggested by Eqs. (1) and (2), and which is derived mathematically in Appendix III, is as follows: If we consider a number of reflectors, all with the same area and the same efficiency, and all having divergence-angle characteristics of the same type, but with different long range specific intensities and divergence limits, then the long range specific intensity will be inversely proportional to the square of the divergence limit, the constant of proportionality depending on the common area, efficiency, and type of divergence-angle characteristic. Through this relation:

(a) We have a precise expression of the principle—already indicated early in Part I—that the effectiveness of a reflector of given area at observation points close to the light source can be increased only at the expense of its effectiveness at points farther out.

(b) The divergence-angle characteristic is expressed in a form (Eq. (4) of Appendix II) which is suitable for use in mathematical investigation of those test methods in which the return light flux is measured.

A complete scheme for describing reflector performance has now been proposed and given such discussion as seemed necessary. Next in order might be an account of reflector specifications—past, present, and prospective future—but for the fact that specifications cannot be considered apart from the test methods by means of which compliance with them is to be judged. This interrelation is particularly close in the present problem, for very special methods must be used to measure reflector performance, involving expense which must be a major consideration in the choice of the specifications themselves. The next paper will therefore describe and compare the various test methods which can be used to measure specific intensity.

E. Mathematical Appendix

The principle underlying each of the three calculations to be given here is that the total amount of light flux in the return beam will in all cases equal the amount of flux received by the reflector, less that which is lost by absorption, extraneous reflection, etc. On a reflector of area A square inches which is subjected to an illumination of E footcandles, there falls a light flux of $EA/144$ lumens. The amount of flux to be accounted for in the return beam is thus $kEA/144$ lumens, where k is the efficiency.

1. If specific intensity is constant within divergence limit

In this case the amount of flux calculated above is distributed uniformly throughout the cone of half-vertex angle D' degrees, which means that it fills a solid angle of $\pi D^2=[2\pi/360]^2\pi D'^2$, where D is the divergence limit in radians, and a very slight approximation has been made.[6] Since candlepower equals light flux per unit solid angle, we obtain for the effective candlepower of the reflector,

$$C'=\frac{(360)^2kEA}{4\times144\pi^3D'^2}=7.3\frac{kEA}{D'^2},$$

and for its specific intensity,

$$I=C'/E=7.3kA/D'^2. \qquad (1)$$

2. If specific intensity follows Gauss "error" curve

The Gauss error function has the mathematical form, $y=ce^{-ax^2}$, where c and a are constants at our disposal and e is the base of natural logarithms, 2.718 . . . . In our problem, y becomes the specific intensity, I, which is to be represented, c the long range specific intensity, I_0, x is the ratio, δ/D, of divergence angle to divergence limit, and a is to be chosen so that $I=0.01\ I_0$ when $\delta=D$. The value of a is thus the natural logarithm of 100, which is 4.6. The expression for specific intensity as a function of divergence angle thus becomes

$$I=I_0e^{-4.6\delta^2/D^2}. \qquad (3)$$

The total amount of light flux in the return beam must now be expressed by an integral, and we have for this flux

$$2\pi E\int_0^\infty I(\delta)\delta d\delta.$$

[6] Throughout this Appendix, D and δ (unaccented) will represent divergence limit and divergence angle in *radians*. Of course, where their ratio, δ/D occurs, it is immaterial what unit is used, so long as the same unit is used for both.

Strictly speaking, the upper limit should be π, but with the function which we shall use for $I(\delta)$, and with divergence limits of only a few degrees at most, our use of ∞ instead of π will make no appreciable difference in the result, and the calculation is simpler. Substituting the right-hand side of Eq. (3) into the above expression for the flux, and equating it to the expression obtained in the first paragraph of this Appendix, we obtain the equation,

$$kEA/144 = 2\pi EI_0 \int_0^\infty e^{-4.65^2/D^2} \delta d\delta = \pi EI_0 D^2/4.6.$$

Solving this equation for I_0 and substituting for D (radians) in terms of D' (degrees), we obtain

$$I_0 = 33.4kA/D'^2, \qquad (2)$$

which enables us to rewrite Eq. (3) as

$$I = \frac{33.4kA}{D'^2} e^{-4.65'^2/D'^2}.$$

3. The general case

We can derive essentially the same relation as Eq. (2) without making any assumption about the relation between specific intensity and divergence angle, except that it is to be of the same form for the various reflectors compared. That is to say, for each reflector,

$$I = I_0 f(\delta/D),$$

where I_0 and D differ from one reflector to another, but not the form of the function f. The calculation is exactly similar to the one just completed, except that we cannot evaluate the integral involved, since it will contain the unknown function f. The important thing is that the value of the integral is still proportional to the square of the divergence limit; it is

$$\int_0^\infty f(\delta/D)\delta d\delta = D^2 \int_0^\infty f(x)x dx,$$

and the integral on the right-hand side depends only upon the form of $f(x)$. We therefore find that in the general case,

$$I_0 = PkA/D'^2,$$

where P is a constant depending upon the form chosen for $f(x)$; if $f(x)$ is constant for x less than unity, $P = 7.3$; if $f(x)$ is the Gauss error function, $P = 33.4$; and the value of P corresponding to any other special assumption can be calculated.

III. Methods of Measuring Specific Intensity

In order that the scheme for describing reflector performance set forth in Parts I and II may be utilized and embodied in specifications, methods for measuring specific intensity are considered. As the first preliminary to description of particular methods, the question of the test distance to be used is discussed. Test distances of 100 feet or greater would be entailed by the presumed inapplicability of the inverse-square law at shorter distances, were it not that a lens can be placed in front of the reflector to make the readings obtained in its principal focal plane characteristic of an infinite test distance. Other preliminary considerations relate to: The illumination attainable at the reflector, the precision required in the measured values (about five percent), the testing of reflectors by groups (preferably avoided), and the smallest specific intensities which would need to be measured in testing reflectors for various classes of service. The particular methods discussed in the light of these considerations are: Illuminometer method, direct visual matching with calibrated comparison lamp, Maxwellian view photometer, and an optical substitution method here proposed as offering important advantages.

IN order to provide a suitable basis for specifications and tests, the writer has developed in the first two papers of this series,[1] a scheme for describing the optical properties of retrodirective[2] reflectors in terms of their specific intensities. The purpose of the present paper is

[1] "I. Fundamental Optical Properties;" "II. General Basis for Optical Descriptions, Specifications, and Tests;" to be referred to as Parts I and II.

[2] A term proposed in Part I, Section A, as being properly descriptive of the kind of reflectors indicated in the general title of these papers.

to consider the laboratory problems involved in the measurement of specific intensities and to survey the methods available for making such measurements.

A. Measurement of Specific Intensity—General Considerations

Laboratory determination of the optical properties of retro-directive reflectors requires the measurement of specific intensity at various settings of divergence and entrance angles, and possibly also of azimuth and orientation angles. If all measurements are made with source and reflector in fixed locations, as is customary and convenient, changes are made in the divergence angle by moving the point of observation transversely with respect to the source-reflector line, and in the entrance angle by tilting or turning the reflector. Azimuth and orientation angles have been so defined that they (as well as the entrance angle) are conveniently controlled by the setting of a suitable reflector mount (see Fig. II—2 of Part II). To determine a specific intensity we must subject the reflector to a known illumination, measure its effective candlepower in a chosen direction, and divide effective candlepower by illumination. Before discussing methods which can be used for measuring the effective candlepower, we shall consider the distance at which the measurement is to be made, the manner in which the reflector is to be illuminated, and the precision which is required.

1. Test distance

Since it is questionable whether the return beam from a retro-directive reflector obeys the inverse-square law at distances less than 100 feet,[3] a test distance at least this great is indicated, and 100 feet is, in fact, the value prescribed several years ago in the I.E.S.-S.A.E. recommended practice for testing "Reflex Reflectors for Motor Vehicles."[4]

However, it is not actually necessary to use such an inconveniently large test distance, for at any given distance, say 20 feet, readings equivalent to those which would be obtained at an infinite distance can be secured by placing in front of the reflector a converging lens (see Fig. III-1) with a focal length of 20 feet, and putting the observation point 20 feet from the lens—thus in the principal focal plane. The effect of the lens is to create in this plane a pattern of light which is exactly similar to the pattern formed at infinity when no lens is used, and which is of such size as to subtend at the center of the lens the same divergence angles as the pattern at infinity would have subtended at the reflector.[5]

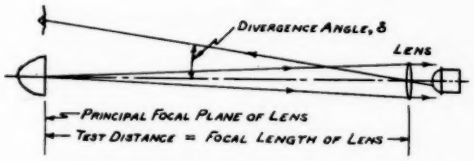

Fig. III-1. Use of lens at short test distance.

Except for losses of light at the lens, the illuminations produced by the reflector at corresponding points of the two patterns are inversely proportional to the squares of their distances, so the same candlepower values would be inferred from each pattern. The pattern formed with the lens in place is thus the same as it would be if the inverse-square law applied at such short distances and the lens were not used. *We can therefore use as short a distance as is convenient, interposing a lens of the proper focal length and correcting for reflection at its faces, and obtain readings characteristic of very great (infinite) distances.* The distance from lens center to point of observation is the distance to be used in determining the divergence angle and in computing effective candlepower from observed illumination. The idea of using a lens in this manner was suggested to the writer by a similar lens application in an apparatus for testing reflectors described by Kingslake;[6] this apparatus

[3] See Part II, Section B.

[4] *S. A. E. Handbook* (Society of Automotive Engineers, New York, 1939), p. 611. Retro-directive reflectors are known to the trade as "reflex reflectors."

[5] In the language of optics: the lens forms in its principal focal plane a real image of the virtual object at infinity.

[6] R. Kingslake, J. Opt. Soc. Am. **28**, 323 (1938). Exactly the same idea was proposed for use in short range photometric measurements on lighthouse beams by A. Blondel in a sealed note deposited with the French Academy of Sciences in 1920 and published in 1929; see A. Blondel, Comptes rendus **188**, 1464 (1929), or Sci. Abs., Abstract No. 3543 (1929). The use of this idea in testing automobile head lamps is described by A. Marsat and P. Cibis, Soc. Franc. Elect., Bull. **8**, 869 (1938); this paper is covered by Sci. Abs., Abstract No. 4922 (1938).

does not measure specific intensities in detail and will be discussed in Part IV.

Extreme reduction of the test distance is still precluded by the fact that the illumination varies rapidly with distance if the pattern is small. At 20 feet, an angle of 2 degrees corresponds to a distance of 8.4 inches, and this angle includes all divergence angles of interest. The points of the pattern corresponding to highway distances from 500 to 1000 feet will be so close to the center that rather great precision will be necessary in locating the observation point and in keeping the observed area small, particularly since specific intensity increases rapidly with decreasing divergence angle for some reflectors, even down to divergence angles as small as 10 minutes. (10 minutes corresponds to 0.70 inch at 20 feet.)

2. Illumination

It is usually advantageous to illuminate the reflector under test as strongly as possible, for its effective candlepower is even then usually so low as to make the measurement rather difficult. Light from a primary source, such as an incandescent lamp filament, must therefore be concentrated on the reflector by means of a lens, a lens system, or a concave mirror. Any such optical system directs a solid cone of light rays at each point of the reflector under test; as illustrated in Fig. III-2, the base of this cone

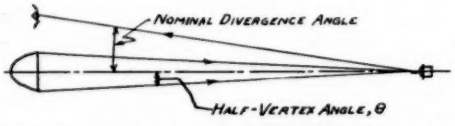

FIG. III-2. Illuminating cone and divergence angle.

covers the entire effective area of the illuminating lens or mirror, which becomes in effect an extended light source. The range of directions thus represented in the illuminating beam creates a corresponding range in the divergence angles at which return light is received at any particular observation point. The divergence angles represented in the light returned to one point will range above and below the average value by an amount equal to the half-vertex angle θ of the illuminating cone, although values close to the average will be emphasized more than extreme ones. The average divergence angle is the nominal one which is measured from the center-line, or axis, of the cone.

There is a fundamental connection between the range of divergence angles which we must tolerate in a single observation and the illumination which we can produce at the reflector, for the illumination which can be produced by a primary source of brightness B when as much of its light as possible is collected in a cone of half-vertex angle θ is given by the equation[7]

$$E=\pi B \sin^2 \theta,$$

where E is in footcandles if B is in candles per square foot. For a tungsten filament at 3000°K (a high operating temperature), $B=1257$ candles/cm$^2=1.2\times10^6$ candles/ft.^{2}. The formula gives in this case about 45 footcandles as the maximum illumination which can be attained with $\theta=0°12'$. This angle corresponds to a lens or mirror 8 inches in diameter at 100 feet, or 1.6 inches at 20 feet. Considering unavoidable losses, we should not expect more than about 20 footcandles under the conditions described.

The value of θ which can be tolerated depends somewhat upon the nominal divergence angle at which the measurement is to be made; in particular, it depends upon the shape of the divergence-angle characteristic in the neighborhood of this divergence angle. If this characteristic curve[8] is practically straight throughout the range of width θ on each side of the nominal divergence angle, no appreciable error will result. No simpler rule which will be safe under all circumstances can be given, but θ probably should never exceed one-half of the nominal divergence angle, and is better kept smaller than one-quarter of that angle. If the same size of illuminating cone is used at all divergence angles, the value of θ might perhaps be properly chosen as one-half the smallest one. If the smallest divergence angle is to be 10 minutes, this will mean an aperture diameter of 3.5 inches at 100 feet, or 0.7 inch at 20 feet. There is a considerable advantage to be gained, however, by using larger cones of light at the larger divergence

[7] See A. C. Hardy and F. H. Perrin, *Principles of Optics* (McGraw-Hill, 1932), p. 409.

[8] We here consider the curve as if plotted to uniform scales, not to a logarithmic scale for specific intensity, as recommended in Part I.

angles, where the specific intensity is lower, for this tends to keep the effective candlepower to be measured at a constant value.

3. Precision

The precision to be demanded of specific intensity measurements depends upon the use for which they are intended. Even such results as are accurate only to within a factor of 2 may be quite valuable in judging different makes of reflectors, for larger variations are common among the brands widely sold, but more accurate results are needed when definite specifications have been set up. The type of specification requirement making the greatest demands on precision of measurement is that prescribing uniformity of the various reflectors in a given lot or set of samples. Such a requirement is appropriate when a number of reflecting units are to be used together, as in a sign, where variation of effective candlepower from one reflector to another would impair the legibility of the text. The writer knows of no satisfactory basis for fixing upon the particular amount of variation which should be considered serious, but certain state highway departments require that the specific intensities (under specified conditions) of all reflectors tested shall be between 75 percent and 125 percent of the average.[9] Such a requirement is more or less in harmony with the writer's estimate in Part I that values which differ by amounts much less than twenty-five percent are practically identical, and indicates that the probable error of the laboratory results should not exceed about five percent.[10]

4. Testing groups of reflectors

In one technique which has been used, 35 reflectors are graded by eye (entrance and divergence angle specified) into 5 groups on the basis of specific intensity, and optical tests are made on groups 1, 3, and 5 as representing the reflectors of maximum, average, and minimum performance, respectively. The 7 reflectors of each of these three groups are mounted in a close hexagonal pattern (one at the center) and tested as a unit. The principal purpose of this procedure is to make the specific intensity of the composite unit high enough for convenient measurement by the method employed. A single set of measurements yields the average properties of seven reflectors, and this may in some respects be an advantage, but the preliminary grouping seems to be of doubtful objectivity and reproducibility. Moreover, the average properties of the middle group are not actually the average values for the whole set of samples, and the difference between corresponding values for the extreme groups is somewhat less than the difference for the extreme individual reflectors. A scheme by which individual reflectors are tested, perhaps not all in the same detail, would seem to be preferable.

B. Methods Available for Measuring Specific Intensity

1. Requirements

The principal difficulties which must be overcome in the measurement of specific intensity arise from the combination of two limitations which have already been described—the limitation to moderate illuminations because of difficulties with divergence angle range, and the necessity of using test distances of the order of 20 feet or more because of the amount of detailed information required. Another essential item is the order of magnitude of the specific intensities which must be measured. Since different methods of measurement may prove to be suitable for reflectors intended for different classes of service, particularly when the reflectors for one class are clear and those for another are colored, we list in Table I the smallest values which would need to be measured for several classes of service. The intention is to indicate approximate magnitudes only, not to say what minimum values should be specified.

If the laboratory illumination at the reflector

[9] This requirement seems to have originated in connection with an early test method (see Harry E. Neal, "Specifications and tests for reflector buttons," American Highways, pp. 12, 13, July, 1936) which can hardly be said to measure specific intensities. The same percentages were carried over in one state to apply to specific intensity measurements, although in some respects they do not have the same significance when referred to the different test methods. This point will be discussed in detail in Part IV.

[10] This conclusion is in general accord with the usual statistical rule that differences more than four times as great as the probable error of the measurements may be considered as significant, and with the fact that the Electrical Testing Laboratories append to their specific intensity data the remark: "The order of accuracy of the candlepower data in this report is approximately 5 percent."

TABLE I.

CLASS OF SERVICE	SMALLEST SPECIFIC INTENSITY PER REFLECTOR UNIT, CP/FT.-C.	AUTHORITY
Highway delineation heads (for posts)	10	Estimate by writer
Clear buttons for signs	0.1	Ohio Dept. of Highways, Supplemental Specification for Reflector Buttons, No. M-110.15, 1938
Red buttons for signs	0.02	Estimate by writer on basis of Ohio specifications for clear buttons. No specifications for colored buttons seem to have been formulated. Red would probably be lowest color.
Red reflectors for trucks and busses	0.05	S.A.E. Recommended Practice
Red reflecting lenses in passenger-car tail lights	0.03	New York State Bureau of Motor Vehicles, Specification, 1935.

is of the order of 20 footcandles, the smallest candlepower to be measured will be about 0.5 cp to about 2 cp, or of the order of 1 cp. We have ignored here the highway delineation heads, which will be about 100 times higher. At a test distance of 20 feet the illumination produced by the return beam at the measuring instrument would be of the order of 1/400, or about 0.002 footcandle, and at 100 ft. it would be only one twenty-fifth as much, or about 0.0001 footcandle.

Now the ordinary method of measuring the candlepower of a light source is to match the illumination which it produces on a diffusely reflecting white surface at a known distance with that produced by a controllable calibrated instrument on another such surface, the two surfaces being viewed simultaneously through a photometer head. The essential feature of this method being a visual match of brightness, its applicability is limited by the sensitivity of the eye, which is such that, even at 20 feet, the illumination from the reflector is too low for sufficiently accurate measurement by this means, while at 100 feet it is so low as to make the use of the ordinary method completely out of the question. It is thus necessary to use some special method, or some variation of the ordinary method, better adapted to low illuminations; the combination of small candlepower and at least moderately large test distance suggests the use of a telephotometric method.

In most telephotometric problems, as for instance in stellar photometry, the dimensions and distance of the source are such that it is effectively a point source, which means that it subtends an angle of less than one minute at the eye, and point-source conditions are essential to the applicability of most telephotometric methods.[11] A reflector having a one-inch aperture would have to be approximately 300 feet away in order to subtend an angle as small as one minute, but the same effect can be secured at shorter distances by using a reducing glass (field glass or telescope, reversed) of sufficient power. For use at 20 feet a glass of at least 15 power will be necessary, or at least 3 power at 100 feet.

2. Illuminometer method

So far as the writer is aware, the only method which has been used to measure specific intensities without employing telephotometric technique is the one now in use by the Ohio Department of Highways for testing reflector buttons.[12] This method employs the ordinary photometric technique referred to above, with a Macbeth illuminometer serving for the illumination measurements, and a test distance of forty feet. The difficulties due to low illuminations are reduced by the scheme of group testing already described, but are no doubt still rather serious. The accuracy of the results is further impaired by the lack of a lens which would make them characteristic of distances at which the inverse-square law can safely be applied. There is, however, a distinct advantage in the fact that the Macbeth illuminometer, which is available commercially, comprises within itself all the calibrated photometric apparatus which is required.

3. Direct visual matching

In some respects the simplest of all, this method utilizes essentially the same technique

[11] For a survey of visual telephotometry, see L. J. Collier, Trans. Illum. Eng. Soc. (London), 3, 141 (1938).

[12] Described in mimeographed material entitled, "Methods of testing reflector buttons, adopted February 18, 1938."

as is employed in stellar photometry when the apparent magnitude of a star is determined by finding a star of known magnitude which has the same appearance, the two being viewed as if side by side by means of a suitable optical arrangement. This is the method which has been used since 1931 for the tests conducted according to the S.A.E. Recommended Practice,[4] where it is described as follows:

"In the laboratory, the reflex reflector to be tested shall be located at a distance of 100 ft. from a single head lamp. The incandescent lamp shall be operated at approximately normal efficiency. The beam candlepower projected on the reflex reflector shall be 10,000 which may be obtained by using a commercial head lamp to which a suitable mask has been fitted. A calibrated standard shall be located adjacent to the reflex reflector and arranged so that the observer can readily vary the candlepower from this standard to make its intensity duplicate that of the reflector under test. The calibrated standard shall be designed to avoid back reflection in the direction of the observer. The calibrated standard shall be of such size and emit light of such color and so diffused that, when viewed by the observer through a reducing glass if necessary, the candlepower from the two units can be readily and accurately compared."

This is the method used by the Electrical Testing Laboratories in obtaining results which are judged to be accurate within ± 5 percent.[10] Projecting 10,000-beam candlepower on the reflector from a test distance of 100 feet will create 1 footcandle illumination at the reflector, so the measured candlepower values are, in fact, specific intensities. It is often more convenient to use a higher or lower illumination in order to bring the effective candlepower within the range of the calibrated standard or to facilitate making the candlepower match, and then to express the results as of 1 footcandle (10,000 cp at 100 feet); this is the actual laboratory practice, which is quite justified, and it emphasizes the appropriateness of the concept of specific intensity.

While this method is basically simple and direct, it is not at all an easy one to use. The essential operation—that of deciding from their appearance when reflector and calibrated standard have the same candlepower in the observer's direction—is fraught with quite serious difficulties, some of which are discussed by Collier.[11] Considerable practice is necessary before consistent results can be obtained, and unsuspected systematic errors may very easily develop. The most fundamental source of trouble lies in the impossibility of viewing the two units simultaneously without favoring one over the other with regard to the sensitivity of the region of the retina on which its image falls. Careful attention must be paid to such matters as dark adaptation of the eye,[13] black background for reflector and standard, observer fatigue, etc. It is necessary even under the best of circumstances to make a number of independent settings, say five, and to average the results. Some rather extensive attempts made under the direction of the writer have so far failed to establish an accuracy of five percent.

Considerably more equipment is required than might be expected. A Macbeth illuminometer or equivalent instrument is necessary for calibrating the standard, checking its calibration at intervals,[14] and measuring the illumination produced at the reflector by the head lamp. Since the candlepower output of an incandescent lamp increases by about 10 percent for a 1 percent increase in lamp current, an ammeter of high precision must be used to measure the electric current through the calibrated standard. For the same reason, both head lamp and standard must be supplied from storage batteries or from a power circuit with specially stabilized voltage. All in all, while the method of direct matching has proved under favorable circumstances to be valid and useful, it leaves much to be desired in the way of speed, economy of apparatus, and easily attained accuracy; it is at least difficult to set up this method so that "the candlepower from the two units can be readily and accurately compared."

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[13] The eye should not be completely adapted to darkness, nor should one stare at the two sources for more than two or three seconds. It is well to have the meters, notebook, etc., fairly well illuminated, and to glance only briefly at the pair of sources, using several glances to find the setting of the calibrated standard which gives the match.

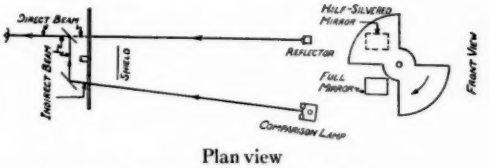
[14] The candlepower output of the standard is varied by changing the current through its incandescent lamp, and under such conditions the stability of its calibration is not as trustworthy as if a constant current were used.

4. Maxwellian-view photometer

The only other recognized telephotometric method which would seem to offer advantages for the measurement of specific intensities utilizes the arrangement called the "Maxwellian view."[15] The essential feature is a converging lens of about 12 inches focal length which receives light from the distant source to be measured. The observer's eye is placed so that the real image which this lens forms of the source falls within the pupil, and the eye is focused on the lens itself. The whole surface of the lens appears bright, tremendously brighter than would a diffusely reflecting surface under equivalent conditions, and this bright field can be matched with a calibrated field by using a photometer head in the usual way. The advantage gained by concentrating in the eye all the light which strikes the lens is more than sufficient to raise the brightness to a level which is easy to match, so far as eye sensitivity is concerned, but there are disadvantages which perhaps more than nullify this gain. In the first place, it is necessary to keep the eye quite accurately in a particular position, and this is difficult and tiring; furthermore, the circumstance that the light passes through only a small portion of the eye lens causes striking and distracting peculiarities in the appearance of the field; finally, the quality of a photometric cube needs to be much higher for use with this arrangement than with a diffuse field. It may be possible to develop a Maxwellian-view photometer which will be satisfactory for specific intensity measurements when used by a highly-trained observer, but the results of the writer's attempts along this line have been discouraging.

5. Optical-substitution telephotometer

The writer has devised an arrangement which avoids the more serious disadvantages of direct visual matching and also permits the inclusion of special features of considerable value in themselves. The schematic diagram of Fig. III-3 shows the arrangement as it would be used with a comparison lamp. The essential features are a



Plan view

FIG. III-3. Optical-substitution telephotometer.

half-silvered mirror and a rotating shutter, whereby the observer sees the reflector and the comparison lamp alternately *in the same apparent location*. During one-half of the cycle (there are two cycles per revolution of the shutter) light from the reflector (the direct beam) passes through the half-silvered mirror to the observer; during the other half, light from the comparison lamp (the indirect beam) is reflected by the full mirror and then by the half-silvered mirror to the observer. Light which would reach the full mirror from the reflector, or the half-silvered mirror from the comparison lamp, is blocked off by the shield.

Using a reducing glass to give reflector and comparison lamp each the appearance of a point source, the observer sees a "star" which changes abruptly in apparent magnitude ("twinkles") four times during each revolution of the shutter, unless reflector and comparison lamp produce equal illuminations at his eye. Only if the losses suffered by the separate beams are exactly equal does this indicate *equality* of candlepower, but the candlepower ratio at which the twinkle disappears is in any case a constant of the instrument, easily determined by calibration. This arrangement possesses the following obvious advantages over the arrangement for direct visual matching:

(a) Images of reflector and comparison lamp fall on the same spot of the retina, thus eliminating the difficulty due to foveal *versus* extra-foveal vision.[11]

(b) The observer is completely unaware of the identities of the two images, and thus cannot be biased in favor of either one, nor can he obtain a spurious consistency in his settings.

(c) There is no possibility of the observer's staring overlong at either "source," and thus becoming subject to a differential fatigue.

Since the precision with which a balance can be made depends upon the ability of the eye to

[15] See A. C. Hardy and F. H. Perrin (reference 7), p. 274; Collier (reference 11); or J. W. T. Walsh, *Photometry* (Constable, London, 1926), p. 109.

detect abrupt changes in apparent candlepower, the feasibility of this method depends essentially upon the sensitivity of the eye to such changes. We of course have at our disposal the speed of rotation of the shutter—each image can be allowed to persist for an appreciable time, the "star" can be made to twinkle more or less rapidly, or a flicker effect can be produced. The writer knows of no published work on this problem, but exploratory investigations carried out under his direction[16] have shown definitely that the accuracy of balance attainable by this method is greater than by direct visual matching of "sources" visible simultaneously, and that the necessity for a trained observer is considerably less. Although further work will be necessary before the most desirable range of eye-illumination, the best accuracy attainable, etc., can be specified, the feasibility of the method for the purposes here considered has been definitely established.

Not only does this arrangement possess the three advantages mentioned above, but it also permits at least two modifications which would not be possible with the simple arrangement for direct visual matching. Because of the spatial separation of the two beams, a variable filter or a pair of Nicol prisms can be introduced into one beam; this permits the comparison lamp to be maintained at constant candlepower, and avoids the variation of color which accompanies changes in lamp current.[17] Also, since this is a null method, a photoelectric cell (or perhaps a photovoltaic cell) and amplifier can be used to indicate the balance point, without introducing difficulties concerning linearity or stability of response. However, the necessity that the spectral sensitivity of the cell be like that of the eye remains as a serious deterrent to this line of development, and the null character of the method even when used visually removes much of the advantage which would otherwise be gained by the use of physical photometry.

[16] By Mr. Lee Showen, in thesis research for the degree of Master of Science in Engineering Physics at the University of Oklahoma.

[17] This possibility also exists for direct visual matching if the half-silvered mirror is used without the shutter, the adjustment being such that the two images appear side by side. The arrangement would then duplicate the essentials of the artificial-star method of stellar photometry.

6. Comparison source energized by illuminating beam

Whatever photometric method is used, a very worthwhile simplification would be achieved if, instead of measuring both the effective candlepower of a reflector and the illumination upon it, and then taking their ratio, one were to determine the ratio more directly. That is to say, since it is the ratio only which is sought, there is poor economy in making absolute measurements of the two quantities separately. An arrangement which would insure a strict proportionality between the candlepower of the comparison source and the illumination at the reflector would make it unnecessary to keep each of these quantities accurately constant while their values at balance are being observed.

This end can be attained by deflecting part of the light out of the illuminating beam by means of a glass plate, and using this light to energize an artificial star[18] incorporated within the instrument. It will be essential in such a case to use a variable filter, or equivalent means, for controlling either the intensity of the light from the reflector or that from the artificial star. There would be a decided advantage to controlling the light from the reflector, since all balances would then be made at a constant apparent candlepower, but in either case the filter setting can be translated directly into specific intensity by means of a calibration curve. Therein lies the great advantage of such a method, for since variations in the light output of the single lamp used will not affect the balance, there will be no need to stabilize voltage, or even to recalibrate when a new lamp is substituted. However, the constancy of the calibration will depend upon the cleanliness of all optical surfaces traversed by the light after passing the beam-splitting plate, so this matter will demand careful attention.

Such a scheme as has just been described in general terms can be applied to the method of direct visual matching, to the Maxwellian view, or to optical substitution. The writer hopes soon to be able to describe in detail its successful use in connection with the last of these methods.

[18] For descriptions of artificial stars, see J. W. T. Walsh (reference 15), p. 424; or L. J. Collier (reference 11).

The Dispersion of Magnetic Double Refraction in the Short Infra-Red Spectrum

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A study has been made of the infra-red dispersion of the magnetic double refraction—Cotton-Mouton effect—in the wave-length region from $\lambda=0.650\mu$ to 1.6μ of the following liquids: nitrobenzene, chlorobenzene, carbon disulfide, halowax oil, *o*-nitrotoluene, benzene and toluene. Ratios of the observed double refraction to the electric double refraction are shown to be constant within experimental error. Comparison of observed dispersion curves with calculated curves from Havelock's law is also shown to fit for the most part within experimental error. This indicates that the magnetic and optical anisotropies of the molecules are relatively independent of the wave-length. The deviation of Havelock's curve from the observed curves in general indicate that Havelock values for short wave-lengths are too high, and for long wave-lengths too low. The deviations shown for the liquids tried are greatest for benzene, where for long wave-lengths the difference amounts to about 5 percent.

THE dispersion of magnetic double refraction has been worked out rather thoroughly in the visible spectrum by various investigators,[1] but apparently no previous work has been done in the infra-red region. It was accordingly thought worth while to extend the dispersion measurements as far as possible into the short infra-red region, or from about 0.627μ to 1.8μ. This has been done for seven liquids and comparison of the results made with theory.

THEORY

The theory of magnetic double refraction follows very closely that of its electric analog, the Kerr electro-optic effect. In most cases the effects have been treated together, merely substituting analogous magnetic constants for the electric constants in the various formulae.

If n_p and n_s are the refractive indices for the components of the light vibrating parallel and perpendicular respectively to the lines of force, the phase difference D in radians after passing through L cm of the substance in a magnetic field of H oersteds is

$$D=\frac{2\pi L(n_p-n_s)}{\lambda}=2\pi CLH^2,$$

where λ is the wave-length of the light. C is called the Cotton-Mouton constant.

Since magnetic double refraction and the Kerr electro-optic effect are so similar, the ratio of C the Cotton-Mouton constant, and the analogous Kerr constant K should be a constant independent of wave-length. This comparison will be shown later.

Havelock[2] has shown that

$$C=\frac{h(n^2-1)^2}{n\lambda},$$

where n is the index of refraction of the substance outside the field and h is a constant independent of the wave-length.

Langevin and Born[3] have worked out a more probable theory which applies primarily to gases in assuming that the molecules are magnetically as well as electrically and optically anisotropic. They have shown that

$$C=\frac{(n^2-1)(n^2+2)}{4n\lambda}\cdot\frac{\Theta_1+\Theta_2}{\alpha_0}\cdot\left(1+\frac{4}{3}S_m\right),$$

where n and λ are as before, α_0 the polarizability, S_m the magnetic susceptibility and

$$\Theta_1=\frac{1}{45kT}[(w_1-w_2)(b_1-b_2)+(w_2-w_3)(b_2-b_3)+(w_3-w_1)(b_3-b_1)],$$

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[1] For general references see J. W. Beams, Rev. Mod. Phys. **4**, 133–172 (1932); G. Szivessy, *Handbuch der Physik* (Julius Springer, 1929), Vol. 21, pp. 724–884.

[2] T. H. Havelock, Proc. Roy. Soc. **A80**, 28 (1907); **A84**, 492 (1911); Phys. Rev. **28**, 136 (1909).

[3] M. Born, Ann. d. Physik **55**, 177 (1918); P. Langevin, Le Radium **7**, 249 (1910); Comptes rendus **151**, 475 (1910); G. Szivessy, *Handbuch der Physik* (Julius Springer, 1929), Vol. 21, pp. 808–823; P. Debye, *Marx Handbuch der Radiologie* (Leipzig, 1923), Vol. 6, p. 769.

$$\Theta_2=\frac{1}{45k^2T^2}[(m_1^2-m_2^2)(b_1-b_2)$$
$$+(m_2^2-m_3^2)(b_2-b_3)+(m_3^2-m_1^2)(b_3-b_1)],$$

where k is the Boltzmann constant, T is the absolute temperature, b_1, b_2, b_3 are the moments induced in a molecule along its three principal axes of optical anisotropy by unit electric force in the light wave acting along the three axes, respectively, w_1, w_2, w_3 are the magnetic moments induced in the molecule by unit magnetic force acting in the same three directions, respectively, and m_1, m_2, m_3 are the components of the permanent magnetic moment along these same three directions. If we assume S_m small and substitute for n^2+2 from the well-known Lorentz-Lorenz relation,[4]

$$\frac{n^2-1}{n^2+2}=\frac{4\pi N\alpha_0}{3}=\frac{4\pi N}{3}\cdot\frac{b_1+b_2+b_3}{3},$$

where N is the number of molecules per cm³, we shall arrive at the Havelock law if the optical and magnetic anisotropies of the molecules do not change with wave-length, or vary in such a way as to give compensation.

Apparatus

The apparatus was fundamentally that used by Ingersoll[5] in measurements of the infra-red dispersion of the Kerr effect. The particular method of measuring ellipticity, used in the present work, has also been described,[6] as well as the selection and testing of the series of mica quarter-wave plates, which are essential to this method.[6,7] The essential differences between this and the Kerr effect work are a different type of cell to hold the liquids and a large electromagnet, which replaced the Kerr cell and the high voltage arrangements. Also a plane mirror was used between the analyzer and the spectrometer with an incident angle of about 60°, so that the apparatus could be arranged more compactly. This introduces no error since the light was transmitted through the various polarizing elements before striking the mirror.

Cell

The cell used (Fig. 1) was made of brass with inside dimensions 8.45×2.45×0.45 cm. After many attempts at cementing glass windows on the ends, the best method found was to use

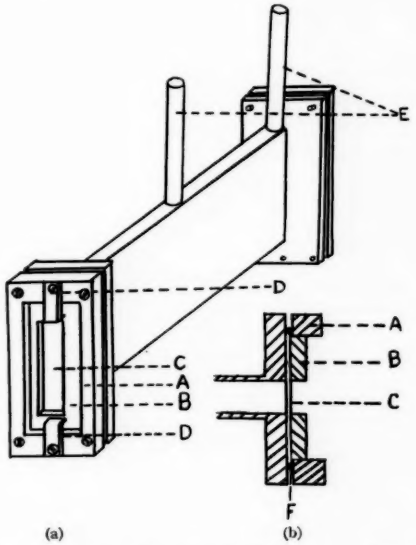

FIG. 1. (a) Cotton-Mouton cell; (b) Cross-sectional view of end of cell as seen from above.

microscope cover glass windows and clamp them by their edges with a rectangular frame A (Fig. 1). This compressed a rubber gasket F (simple elastic band) against the edges of the cover glass in such a way as to prevent leakage and still introduce no strain. One of the small brass tubes E was for filling purposes and the other for holding a thermometer.

Magnet

The large electromagnet was of over-all dimensions about 100×150×70 cm. The windings were made of $\frac{3}{16}$-inch insulated copper tubing through which distilled water was pumped for cooling purposes. The distilled water in turn was cooled by an exchanger which circulated lake water at about 5°C around a section of the

[4] P. Debye, *Marx Handbuch der Radiologie* (Leipzig, 1925), Vol. 6, pp. 754–776; L. Lorenz, Ann. d. physik. Chemie **11**, 70 (1880); H. A. Lorentz, *Theory of Electrons* (Leipzig, 1909), p. 144.

[5] L. R. Ingersoll and Wesley R. Winch, Phys. Rev. **44**, 399–405 (1933).

[6] L. R. Ingersoll, J. Opt. Soc. Am. **27**, 411–413 (1937).

[7] Ingersoll, Rudnick, Slack and Underwood, Phys. Rev. **57**, 1146 (1940).

distilled water circuit. The rate of flow of lake water could be used to control the temperature of the magnet and consequently the cell containing the liquid, which was in thermal contact with the magnet pole pieces. The pole pieces were 12 cm in diameter but tapered down to 5.1 cm in diameter at the cell where they were 6 mm apart.

The field was kept within narrow limits by keeping the current (approximately 50 amperes) constant within 0.3 percent, resulting in a variation of the magnetic field much less than this. The magnet was constructed to carry 150 amperes, but the field used was adequate and the temperature much easier to control. The magnet was mounted so as to be able to rotate around a vertical axis. This was used as discussed later to eliminate the Faraday effect. The field used, as measured by a Grassot fluxmeter using a flip coil, was approximately 26,100 oersteds.

Sources of Error

Impurities

Measurements of the Cotton-Mouton effect are less influenced by impurities than the Kerr electro-optic effect, due to the absence of possible conducting currents producing heating effects, etc.; however, the best available material was used. Some of the liquids (nitrobenzene, chlorobenzene, halowax) were the identical ones used in measurements of the infra-red dispersion of the Kerr electro-optic effect[5] which originally had been very carefully purified by distillation and other treatment. Purest obtainable benzene (Eastman), *o*-nitrotoluene (Eastman), toluene (Eastman) and carbon disulfide (Mallinckrodt) were used untreated. The Cotton-Mouton constants as measured for these liquids were found to compare as well as could be expected with values from the *International Critical Tables*.

TABLE I. *Relative values of Cotton-Mouton constant for different substances used.* $CS_2 = -100$.

Liquid	Observed C $=0.627\mu$	I.C.T. $=0.578\mu$
Nitrobenzene	467	510
Chlorobenzene	138	147
Carbon disulfide	−100	−100
Halowax oil	570	—
o-nitrotoluene	292	296
Benzene	120	119
Toluene	132	125

Temperature

The temperature of the liquid in the cell was read at intervals of 15 minutes and was kept at 20°C within 1°C. Corrections were applied for these variations even though they were very slight. The Cotton-Mouton constant varies approximately as $1/T$, therefore these corrections were of the order of 1 part in 300 which is somewhat less than the experimental error in the present measurements.

Strains in glass

Any strain in glass such as in the end plates of the cell will introduce a phase difference which complicates measurements of magnetic double refraction. This is one of the reasons why cementing the cover glasses on the cell was not desired, as the cements tried introduced strains that produced phase differences which were several times that due to the liquid.

However, any *small* phase difference due to residual strains in glass end plates, or any other cause such as may occur from reflection not normal to mirrors, was automatically eliminated. The deflection of the galvanometer, which was proportional to the phase difference, was obtained by readings with the magnetic field on and off, and therefore only measured the additional phase difference due to the magnetic field. The empty cell was also tried for possible magnetic double refraction due to the glass end plates and this was found imperceptible.

Faraday effect

This error could arise from two different causes: (1) that due to the conical beam and (2) that due to not having the magnetic field perpendicular to the light beam.

Considerations of the first will show that if the axis of the conical beam is perpendicular to the field the sum of all the components of the field along the different rays making up the conical beam will be zero so the net effect will not affect the readings.

The possibility of any error due to Faraday effect from not having the magnetic field exactly perpendicular to the beam of the light, as well as

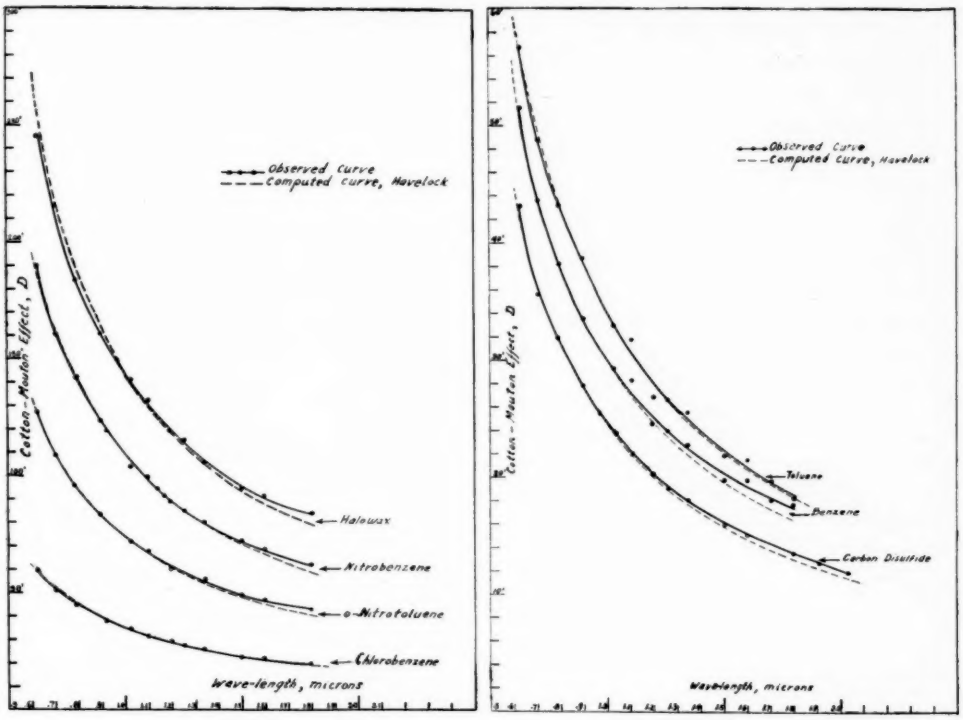

FIG. 2. (a) Cotton-Mouton dispersion curves for halowax, nitrobenzene, *o*-nitrotoluene and chlorobenzene; (b) Cotton-Mouton dispersion curves for toluene, benzene and carbon disulfide.

the conical beam, was eliminated by removing the mica quarter-wave plate. When the quarter-wave plate was removed the galvanometer deflection was directly proportional to the Faraday rotation. The magnet was then rotated about its vertical axis until no perceptible effect was noticed. If there was a small rotation due to the Faraday effect present it was found it still did not introduce any error in measurement of phase difference, since it averaged out when the field was reversed. That is, the galvanometer deflection due to double refraction was in the same direction on reversal of the field, since the sign of the ellipticity was the same in both cases. In the Faraday effect, however, the rotation is reversed with reversal of the field, and would reverse the galvanometer deflection. Therefore readings were always taken with the field in both directions.

Spectral impurity

There was an inevitable coverage by receiver (and slit image) of an appreciable spectral range. This amounted to about 0.01μ at $\lambda=0.6\mu$ to 0.10μ at $\lambda=1.6\mu$. The error due to this becomes quite evident near an absorption band. It was therefore deemed necessary to determine transmission curves on all the liquids used. These are not shown but the curves for all the substances tried except CS_2 are very similar with absorption bands centering approximately at $\lambda=1.14\mu$ and 1.42μ. The experimental points that are affected the most by these absorption bands are those at $\lambda=1.1\mu$ and 1.19μ.

In drawing the accepted curve recognition was made of this fact although the points shown are those indicated by the spectrometer. In the case of carbon disulfide there are no absorption bands in this infra-red region, and it is noted

that the experimental points fall much nearer to the accepted curve. While the error due to this could be theoretically calculated the points involved were few and it was thought that "smoothing the curve" was a better way of taking care of this error than could be done by the somewhat uncertain methods of calculation.

Results

Table I shows relative values of the Cotton-Mouton constant for the liquids tested as observed at $\lambda = 0.627\mu$ which was the shortest wave-length observed. Also in the table are shown the values taken from the *International Critical Tables* for $\lambda = 0.578\mu$. Both I.C.T. and observed values are relative to carbon disulfide which is taken as -100. All values agree as well as can be expected under the circumstances.

The actual phase difference observed is given in Fig. 2. All the liquids were used under the same conditions except nitrobenzene, which was taken at a slightly smaller field strength. If the values for nitrobenzene are multiplied by 1.06, the phase differences can be compared with the others.

Table II shows the relative Cotton-Mouton constants for various wave-lengths referred to that at wave-length 1 micron as unity. Carbon disulfide is the only one of these liquids which shows a negative value for this constant. The ratio of the Cotton-Mouton constants to the Kerr constants is shown to be constant within experimental error, except possibly for carbon disulfide at the longer wave-lengths. This shows that the same type of theoretical formula will satisfy the dispersion of both electric and magnetic double refraction very closely. The Kerr constants used are those obtained by Ingersoll.[5]

Computed Havelock values adjusted to fit the experimental values at 1 micron as well as deviations of the Havelock values from observed values, are also given in Table II. These will be discussed in more detail for each liquid. For a very simple dispersion formula $D = 1/\lambda$ is shown to fit experimental data surprisingly well, especially for the longer wave-lengths beyond $\lambda = 1\mu$. The indices of refraction are the values determined by Ingersoll[5, 8] except for carbon disulfide, which are taken from the L. and B. tables.

The experimental points (Fig. 2) at 1.7μ and 1.8μ for all the liquids except CS_2 should have much less weight in determining the curve, since the energy transmitted was very small and the

[8] L. R. Ingersoll, J. Opt. Soc. Am. and Rev. Sci. Inst. **6**, 663 (1922).

TABLE II. *Relative values of Cotton-Mouton constant C for different wave-lengths. K = Kerr constant.*

λ	n	Observed C	Computed Havelock	$1/\lambda$	C/K	Computed minus Observed
Nitrobenzene						
0.65μ	1.5455	1.651	1.635	1.539		−0.016
0.70	1.5417	1.500	1.497	1.428	1.020	−0.003
0.80	1.5355	1.291	1.279	1.250	1.010	−0.011
0.90	1.5318	1.123	1.120	1.111	0.999	−0.003
1.00	1.5296	1.000	1.000	1.000	1.000	0.000
1.10	1.5282	0.902	0.901	0.909	1.001	−0.001
1.20	1.5271	0.824	0.825	0.833	1.006	+0.001
1.30	1.5261	0.759	0.759	0.769	1.010	0.000
1.40	1.5253	0.705	0.702	0.714	1.010	−0.003
1.50	1.5248	0.663	0.654	0.667	1.000	−0.009
1.60	1.5240	0.631	0.612	0.625	1.008	−0.019
Chlorobenzene						
0.65	1.5193	1.608	1.645	1.539		+0.037
0.70	1.5161	1.468	1.496	1.428	1.014	+0.028
0.80	1.5111	1.251	1.283	1.250	1.001	+0.032
0.90	1.5085	1.105	1.129	1.111	1.004	+0.024
1.00	1.5069	1.000	1.000	1.000	1.000	0.000
1.10	1.5055	0.912	0.912	0.909	1.012	0.000
1.20	1.5044	0.832	0.833	0.833	1.017	+0.001
1.30	1.5037	0.764	0.766	0.769	1.017	+0.002
1.40	1.5033	0.708	0.710	0.714	1.016	+0.002
1.50	1.5029	0.660	0.662	0.667	0.995	+0.002
1.60	1.5025	0.623	0.620	0.625	0.995	+0.003
Carbon disulfide						
0.65	1.6188	1.650	1.653	1.539		+0.003
0.70	1.6138	1.496	1.510	1.428	0.987	+0.014
0.80	1.6060	1.290	1.289	1.250	1.001	+0.001
0.90	1.6006	1.132	1.123	1.111	1.007	−0.009
1.00	1.5969	1.000	1.000	1.000	1.000	0.000
1.10	1.5942	0.897	0.899	0.909	1.001	+0.002
1.20	1.5922	0.824	0.818	0.833	1.010	−0.006
1.30	1.5905	0.758	0.751	0.769	1.012	−0.007
1.40	1.5891	0.705	0.694	0.714	1.014	−0.009
1.50	1.5880	0.656	0.645	0.667	1.013	−0.011
1.60	1.5870	0.620	0.602	0.625	1.031	−0.018
1.70	1.5861	0.582	0.565	0.588	1.029	−0.017
1.80	1.5852	0.549	0.531	0.555	1.020	−0.018
1.90	1.5845	0.522	0.503	0.526	1.023	−0.019
2.00	1.5840	0.496	0.479	0.500	1.022	−0.017
Halowax oil						
0.65	1.6298	1.614	1.660	1.539		+0.046
0.70	1.6251	1.462	1.522	1.428	0.988	+0.060
0.80	1.6183	1.248	1.302	1.250	0.977	+0.054
0.90	1.6132	1.110	1.136	1.111	0.987	+0.026
1.00	1.6098	1.000	1.000	1.000	1.000	0.000
1.10	1.6073	0.903	0.902	0.909	1.001	−0.001
1.20	1.6054	0.821	0.821	0.833	0.993	0.000
1.30	1.6042	0.758	0.755	0.769	0.994	−0.003
1.40	1.6031	0.705	0.698	0.714	0.991	−0.007
1.50	1.6023	0.662	0.649	0.667	1.006	−0.013
1.60	1.6015	0.626	0.607	0.625	1.014	−0.019

error necessarily high. Also the point at $\lambda = 0.627\mu$ has a higher probable error than the others.

Nitrobenzene shows the computed Havelock values to be somewhat less for both short and long wave-lengths, giving a somewhat flatter curve. The deviations, however, are almost within experimental error, which was taken to be about 1 percent.

Chlorobenzene shows that for short wave-lengths Havelock values seem to be too high. However, for wave-lengths longer than 1μ Havelock values fit the observed curve within experimental error.

Carbon disulfide agrees quite well for short wave-lengths. Beyond 1μ the Havelock values gradually fall below the observed curve.

Halowax oil has the largest deviation for small wave-lengths of the liquids tested. The Havelock values are too high for short wave-lengths and too low for long wave-lengths, or give a somewhat steeper curve. This liquid also shows the largest double refraction of those used.

o-Nitrotoluene agrees within experimental error for short wave-lengths; however, the Havelock values fall gradually below for longer wave-lengths, as with carbon disulfide.

Benzene also agrees well for short wave-lengths, but for long wave-lengths deviates more than any of the others tried.

Toluene, similar to chlorobenzene, shows Havelock values somewhat high for short wave-lengths, and fair agreement with long wave-lengths, with the possible exception of $\lambda = 1.6\mu$.

Colloidal iron produced by arcing iron electrodes under carbon tetrachloride and filtering was also tried. However, the double refraction observed was too small (less than a phase change of one minute at $\lambda = 1.1\mu$) to run a dispersion curve of sufficient accuracy to mean anything.

TABLE II.—*Continued.*

λ	n	OBSERVED C	COMPUTED HAVELOCK	$1/\lambda$	C/K	COMPUTED MINUS OBSERVED
o-nitrotoluene						
0.65μ	1.5390	1.638	1.630	1.539		−0.008
0.70	1.5357	1.485	1.494	1.428	0.995	+0.009
0.80	1.5305	1.274	1.272	1.250	0.998	−0.002
0.90	1.5269	1.122	1.122	1.111	1.002	0.000
1.00	1.5244	1.000	1.000	1.000	1.000	0.000
1.10	1.5223	0.907	0.901	0.909	1.002	−0.006
1.20	1.5208	0.830	0.822	0.833	1.009	−0.008
1.30	1.5198	0.750	0.751	0.769	0.985	0.001
1.40	1.5191	0.710	0.699	0.714	1.002	−0.011
1.50	1.5186	0.663	0.653	0.667	0.998	−0.010
1.60	1.5181	0.625	0.610	0.625	1.011	−0.015
Benzene						
0.65	1.4938	1.630	1.615	1.539		−0.005
0.70	1.4908	1.477	1.481	1.428		+0.004
0.80	1.4869	1.270	1.275	1.250		+0.005
0.90	1.4842	1.116	1.120	1.111		+0.004
1.00	1.4823	1.000	1.000	1.000		0.000
1.10	1.4809	0.918	0.904	0.909		−0.014
1.20	1.4797	0.844	0.824	0.833		−0.020
1.30	1.4788	0.781	0.758	0.769		−0.023
1.40	1.4780	0.727	0.701	0.714		−0.026
1.50	1.4774	0.680	0.653	0.667		−0.027
1.60	1.4769	0.646	0.611	0.625		−0.035
Toluene						
0.65	1.4899	1.594	1.607	1.539		+0.013
0.70	1.4874	1.452	1.477	1.428		+0.025
0.80	1.4839	1.256	1.273	1.250		+0.017
0.90	1.4814	1.113	1.120	1.111		+0.007
1.00	1.4796	1.000	1.000	1.000		0.000
1.10	1.4783	0.906	0.904	0.909		−0.002
1.20	1.4773	0.827	0.825	0.833		−0.002
1.30	1.4765	0.759	0.759	0.769		0.000
1.40	1.4757	0.703	0.702	0.714		−0.001
1.50	1.4751	0.658	0.654	0.664		−0.004
1.60	1.4747	0.620	0.612	0.625		−0.008

CONCLUSIONS

The important conclusion to be drawn from this work is the fact that the Havelock equation represents the dispersion of the magnetic birefringence for these seven liquids within about the limit of experimental error for the main part of the spectral region investigated. The average deviation of the experimental from theoretical curves is about 1 percent, and the maximum deviation for all the liquids tried is less than 6 percent in the spectral range $\lambda = 0.65\mu$ to 1.6μ. In the visible, various investigators[9] have also found the experimental dispersion curves to follow the Havelock law within experimental error for all the liquids tried.

In general the substances which show the largest departure from computed values are of the type in which the Havelock formula gives a dispersion curve steeper than the observed one. In several of the curves there seems to be an indication that the observed curve on the extreme short wave-length end will be steeper

[9] A. Cotton and H. Mouton, Comptes rendus **147**, 193 (1908); **150**, 857 (1910); J. de phys. et rad. **5**, 1, 5 (1911); Ann. de Chemie et Phys. **8**, 20, 213 (1910); C. A. Skinner, Phys. Rev. **29**, 541 (1909); G. Szivessy, Zeits. f. Physik **18**, 97 (1923).

than the Havelock curve, but we must bear in mind that here the probable error is higher than throughout the middle of the curve.

In the infra-red dispersion of the Kerr electro-optic effect Ingersoll[5] has shown that nitrobenzene, chlorobenzene, *o*-nitrotoluene and halowax have dispersion curves that are somewhat less steep than the computed Havelock curve; CS_2 was found to fit Havelock's formula over the spectral range tried within experimental error. Except for CS_2 the same is found true here.

The fact that the Havelock formula fits the experimental data so well means, if we assume the Langevin-Born theory, that the quantities Θ_1, Θ_2 and α_0 must be relaitvely independent of the wave-length, or what is less probable, compensate for each other.

I wish to make grateful acknowledgment to Professor L. R. Ingersoll for his suggestion of the problem and invaluable advice given generously. I am also indebted to the Wisconsin Alumni Research Foundation for assistance.

The Refractive Index of Silicate Glasses as a Function of Composition*

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The refraction per gram atom of oxygen, defined by a formula of either the Gladstone-Dale, the Newton, or the Lorentz-Lorenz type, equals the sum of terms characteristic of the component "metallic" elements, each of these terms being proportional to the number of atoms of that element per atom of oxygen. This is shown to be true (with but little error) for practically all classes of silicate glasses for which adequate data are available for testing the relationship. (Deviations from strict proportionality are, however, observed for glasses containing large amounts of lead.) Constants are derived empirically for use in the Gladstone-Dale and Newton type equations. Different constants are needed for triangularly surrounded and for tetrahedrally surrounded boron atoms. Using the equations previously derived for the calculation of the density and volume per oxygen, it is now possible to compute the refractive index (for the sodium D lines) of a wide variety of silicate glasses from the composition alone.

INTRODUCTION

IT was shown in a recent paper[1] on the density of glasses that the volume per gram atom of oxygen, in a well-annealed glass, is given quite accurately by an additive function of the form

$$V_O = k + b_{Si} + c_{Si}N_{Si} + \sum{}' c_M N_M, \qquad (1)$$

where k is a very small constant characteristic of the annealing technique; N_{Si} and N_M are the numbers of silicon or other "metal" atoms per atom of oxygen; b_{Si} and c_{Si} are constants, different for different ranges of N_{Si} values; and c_M are constants, characteristic of the other "metallic" elements, valid (insofar as can be determined from present data) for all values of N_M.

V_O and N_M are related to the density (ρ) of the glass and to the formulas (M_mO_n), weight fractions (f_M), and molecular weights (W_M) of the component oxides by the equations

$$V_O = \frac{1}{\rho \sum n_M f_M / W_M} \qquad (2)$$

and

$$N_M = \frac{m_M f_M / W_M}{\sum n_M f_M / W_M}. \qquad (3)$$

In this paper, it is shown that the refraction, R_O, per gram atom of oxygen obeys an even simpler additive law:

$$R_O = \sum R_{O,M} = \sum a_M N_M. \qquad (4)$$

Making use of these equations, densities and refractive indices of well-annealed glasses can now be readily calculated from the compositions.

GENERAL METHOD. DETERMINATION OF THE CONSTANTS

Using a Gladstone-Dale[2] type of equation, the refraction per gram atom of oxygen is given by the relation

$$R_O = (n_D - 1)V_O = \frac{n_D - 1}{\rho \sum n_M f_M / W_M}. \qquad (5)$$

Here n_D is the refractive index for the sodium D lines.

In theoretical optics, it has been much more customary to use equations of the Lorentz-Lorenz[3,4] form, according to which

$$R_O'' = \left(\frac{n_D^2 - 1}{n_D^2 + 2}\right) V_O = \frac{(n_D^2 - 1)/(n_D^2 + 2)}{\rho \sum n_M f_M / W_M}. \qquad (6)$$

Kurtz and Ward,[5] however, favor equations of the Newton[6] (or Sellmeier-Drude) type—in the present instance

$$R_O' = (n_D^2 - 1)V_O = \frac{n_D^2 - 1}{\rho \sum n_M f_M / W_M}. \qquad (7)$$

R. W. Wood,[7] in his treatment of dispersion, also

* Communication No. 770 from the Kodak Research Laboratories.

[1] M. L. Huggins, J. Opt. Soc. **30**, 420 (1940).

[2] J. H. Gladstone and T. P. Dale, Trans. Roy. Soc. (London) **153**, 317, 337 (1863).

[3] H. A. Lorentz, Ann. d. Physik **9**, 641 (1880).

[4] L. Lorenz, Ann. d. Physik **11**, 70 (1880).

[5] S. S. Kurtz, Jr. and A. L. Ward, J. Frank. Inst. **222**, 563 (1936); **224**, 583 (1937).

[6] Sir Isaac Newton, *Opticks* Book II (1717). See reference 5.

[7] R. W. Wood, *Physical Optics* (Macmillan, New York, 1934).

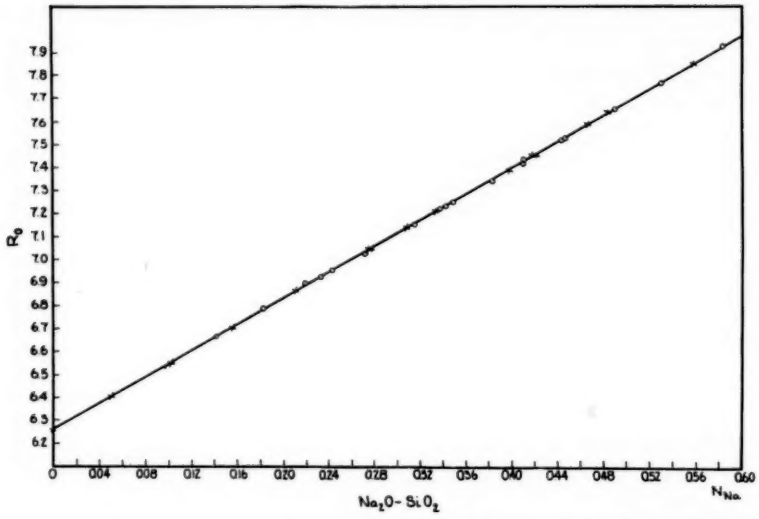

FIG. 1. Na_2O-SiO_2. ○ Faick and Finn; × Morey and Merwin. $R_O = 6.255 + 2.878 N_{Na}$.

uses the $(n_D^2 - 1)$ type of equation. The additivity equation, for R_O' values calculated according to Eq. (7), may be written as follows:

$$R_O' = \sum R_{O,M}' = \sum a_M' N_M. \qquad (8)$$

Still other types of equation, such as those due to Eykman[8] and Lichtenecker,[9] have been proposed and used. Morey and Merwin,[10,11] comparing the deviations from additivity of the specific refractivity of $Na_2O - CaO - SiO_2$ glasses, calculated according to the Gladstone-Dale, Lorentz-Lorenz, Eykman and Lichtenecker formulas, concluded, "While the general pattern of departures is the same for all the formulas tested, the Gladstone-Dale formula gives departures that are little more than one-half as large as the others plotted, and about one-third those of the Lichtenecker formula."

Using data from Morey's book,[11] *The Properties of Glass*, we have made a large number of calculations, both using Eqs. (5) and (4) and using Eqs. (7) and (8), as well as some using Eq. (6) and the corresponding additivity equation. For comparison, using the accurate data[10] of Morey and Merwin and of Faick and Finn on the $Na_2O - SiO_2$ glasses, the average differences between observed and calculated refraction and index of refraction values are given in Table I.

After due consideration of these results, together with Morey and Merwin's conclusions, the fact that Eq. (5) is simpler to use than (7), and (7) is simpler than (6), and the fact that Eqs. (6) and (7) seem to be sounder theoretically than Eq. (5), we decided to use *both* Eqs. (5) and (7). We have therefore deduced two separate sets of constants, a_M and a_M', for use with the additivity Eqs. (4) and (8).

Plotting R_O against N_{Na} for the sodium silicate glasses, it is evident (Fig. 1) that the Morey and Merwin and the Faick and Finn data are in quite good agreement. Least-square solutions of the

TABLE I. *Average deviations between observed and calculated refractions and indices of refraction for sodium silicate glasses.*

	Refraction (R_O)		Index of Refraction (n_D)	
	Faick and Finn	Morey and Merwin	Faick and Finn	Morey and Merwin
Using Eq. (3)	0.0042	0.0060	0.00029	0.00041
Using Eq. (4)	0.0041	0.0069	0.00050	0.00085
Using Eq. (5)	0.0099	0.0211	0.00023	0.00048

[8] J. F. Eykman, Rec. trav. chim. **14**, 185, 201 (1895).
[9] K. Lichtenecker, Physik. Zeits. **27**, 115, 833 (1926).
[10] G. W. Morey and H. E. Merwin, J. Opt. Soc. **22**, 632 (1932).
[11] G. W. Morey, *The Properties of Glass* (Reinhold, New York, 1938).

two sets of data give slightly different equations, however:

Morey and Merwin: $R_O = 6.253 + 2.892 N_{Na}$, (9)

Faick and Finn: $R_O = 6.265 + 2.844 N_{Na}$. (10)

From these results and the relation[1]

$$\sum \frac{n_M}{m_M} N_M = 1, \quad (11)$$

we have deduced the a_{Si} and a_{Na} values in Table II. The a'_{Si} and a'_{Na} values in Table III were obtained similarly, using Eqs. (7) and (8) instead of (5) and (4).

The difference between the Morey and Merwin constants and the Faick and Finn constants, though not large, is sufficient to make it appear advisable to obtain and use two sets of constants throughout, when dealing with the more accurate determinations.

TABLE II. *Values of a_M for use in Eq. (4).*

M	$a_{M(G)}$	$a_{M(B)}$			
	Geophysical Laboratory	National Bureau of Standards	Univ. of Sheffield	Peddle	Others
Si	12.506	12.530	(12.530)	(12.530)	(12.530)
Li	4.4				
Na	6.019	5.976	(5.976)	(6.0)	(5.976)
K	9.53		9.5	9.5	(9.5)
Be					5.9 Becker; Lai and Silverman
Mg			(8.6)		8.6 Larsen
Ca	12.64	12.55	12.55	12.55	(12.55)
Sr					16. Eskola
Ba				20.5	
Zn			12.2		
Pb_{II}	30.			30.	
B′			7.8		
B″			8.8		
Al		10.45	10.5		10.5 Larsen
Bi_{III}					34. Riegel and Sharp
Ti_{IV}			25.		

Applying these Morey and Merwin constants to other data from the Geophysical Laboratory, corresponding constants for K, Ca, and Pb have been obtained. Similarly, using the Faick and Finn values for a_{Si} and a_{Na}, other data from the National Bureau of Standards have yielded a values for Ca and Al. Either of these two sets of constants seems to be applicable, within the probable experimental error, to the data of Turner and co-workers at the University of Sheffield. The additional a values in column 4 of Table II were obtained with the aid of the National Bureau of Standards values (column 3) for a_{Si} and a_{Na}. For simplicity, these have also been assumed in the computations for glasses from other laboratories (columns 5 and 6). The a' values in Table III were obtained in an analogous manner.

TABLE III. *Values of a'_M for use in Eq. (8).*

M	$a'_{M(G)}$	$a'_{M(B)}$			
	Geophysical Laboratory	National Bureau of Standards	Univ. of Sheffield	Peddle	Others
Si	30.84	30.98	(30.98)	(30.98)	(30.98)
Li	12.				
Na	15.54	15.35	(15.35)	(15.35)	(15.35)
K	24.4		24	24.	(24.)
Be					16.2 Becker; Lai and Silverman
Mg			(23)		23. Larsen
Ca	35.00	34.58	34.6	34.6	(34.58)
Sr					44 Eskola
Ba				56	
Zn			34		
Pb_{II}	85			85	
B′			20.0		
B″			21.2		
Al		27.1	27		27 Larsen
Bi_{III}					101 Riegel and Sharp
Ti_{IV}			70		

The graphs from which the constants of Table II were obtained are reproduced in Figs. 2–26. In each case the deviation of a plotted point from the straight lines gives the absolute magnitude of the difference between the R_O value calculated from the published density and composition values by means of Eq. (5) and that obtained on the assumption of strict additivity (Eq. 4), using the constants of Table II.

A similar set of graphs of $R'_{O,M}$ *vs.* N_M was prepared and used to obtain the constants in Table III. Since these graphs differ but little from the $R_{O,M}$ graphs shown here, as regards general appearance and deviations from the straight-line relationships, they are not reproduced.

Breaks, such as have been found by Faick and Finn[12] in the curves representing the change of refractive index with percentage of SiO_2, do not occur with the present method of calculation. This shows that the observed breaks are at-

[12] C. A. Faick and A. N. Finn, J. Am. Ceram. Soc. **14**, 518 (1931).

tributable entirely to irregularities in the *density vs.* composition relation.

It should be noted that the data by Peddle are probably less reliable than most of the others, the composition being deduced from the weights of the component compounds used in making up the glass, rather than from analyses.

The $R_{O,Pb}$ and $R'_{O,Pb}$ values for high values of N_{Pb} are slightly larger than corresponds to a strictly linear relation (Fig. 17). Perhaps, if data

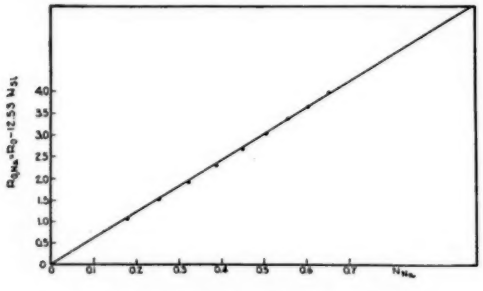

FIG. 2. Na_2O-SiO_2 (Peddle). $R_{O,Na}=6.0N_{Na}$.

were available for other systems over a corresponding range, this would also be found to be true for other elements. There seem to be indications, in some of the other graphs, that this is the case. It is really surprising that the $R_{O,M}$ (and $R'_{O,M}$) *vs.* N_M curves are as linear as they are.

The constant values 30 and 85, given for a_{Pb} and a'_{Pb} in the tables, may be used for the values of N_{Pb} less than, say, 0.2. The deviations for larger N_{Pb} values will be more fully discussed in a later paper.

As in the calculation of densities, it has been found necessary to consider two kinds of boron atoms: one kind (B′) being tetrahedrally surrounded by 4 oxygen atoms, the other (B″), triangularly surrounded by 3 oxygens. Concerning the calculation of the numbers $N_{B'}$ and $N_{B''}$, see the earlier paper[1] of this series.

In the absence of density data, the refractive index of a glass can be calculated, as shown in the next section, from its composition alone, provided the a_M (or a'_M) and c_M constants are known. Similarly, any one of these a_M (or a'_M) constants can obviously be calculated from a set of refractive index and composition data, without densities, provided the others are known [Eqs. (1), (5), and (4), or (1), (7), and (8)]. This method has been used to obtain the Li constants in Tables II and III.

Approximate values for $a_{M(G)}$ constants (Table II, column 2) can be readily deduced from the corresponding $a_{M(B)}$ constants (columns 3, 4, 5, 6) in the following way: From Eq. (4),

$$R_O=\sum a_{M(G)}N_M=\sum a_{M(B)}N_M. \qquad (12)$$

Hence,

$$a_{M(G)}-a_{M(B)}=\sum (a_{M'(B)}-a_{M'(G)})\frac{N_{M'}}{N_M}, \qquad (13)$$

M′ representing each of the component metallic elements (including Si) other than M. For a binary silicate glass, this reduces to

$$a_{M(G)}-a_{M(B)}=(a_{Si(B)}-a_{Si(G)})\frac{N_{Si}}{N_M}. \qquad (14)$$

From the electrical neutrality relation[1]

$$\sum \nu_M N_M=2, \qquad (15)$$

where ν_M is the valence of the element M, one obtains

$$N_{Si}=\frac{1}{2}-\frac{\nu_M}{4}N_M. \qquad (16)$$

Substituting into Eq. (14) yields

$$a_{M(G)}-a_{M(B)}=(a_{Si(B)}-a_{Si(G)})\left(\frac{1}{2N_M}-\frac{\nu_M}{4}\right) \qquad (17)$$

$$=0.024\left(\frac{1}{2N_M}-\frac{\nu_M}{4}\right)$$

$$=\frac{0.012}{N_M}-0.006\nu_M. \qquad (18)$$

Insertion in Eq. (18) of the valence ν_M and the average value of N_M for the series of binary glasses

TABLE IV. *Comparison of refraction constants for silicate glasses and for oxide crystals.*

M	$a_{M(B)}$ (GLASS)	a_M (CRYST. OXIDE)
Si	12.53	12.4 quartz; 12.5 tridymite
Mg	8.6	8.3
Ca	12.55	13.9
Sr	16.	17.9
Ba	20.5	25.0
Zn	12.2	14.7
Al	10.5	9.9

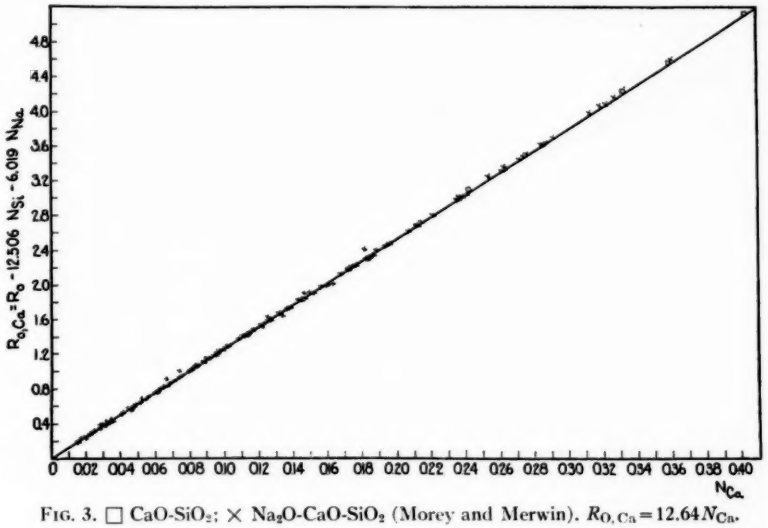

FIG. 3. □ $CaO\text{-}SiO_2$; × $Na_2O\text{-}CaO\text{-}SiO_2$ (Morey and Merwin). $R_{O,Ca}=12.64N_{Ca}$.

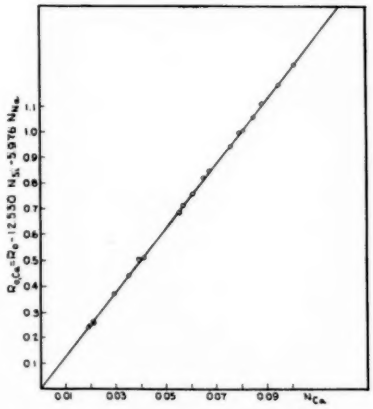

FIG. 4. $Na_2O\text{-}CaO\text{-}SiO_2$ (Faick and Finn). $R_{O,Ca}=12.55N_{Ca}$.

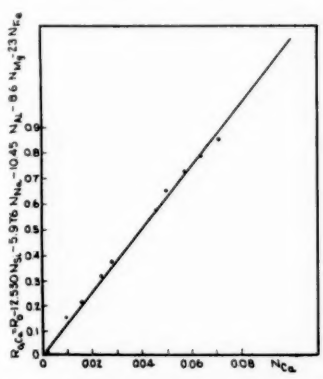

FIG. 5. $Na_2O\text{-}CaO\text{-}SiO_2$ (-$MgO\text{-}Al_2O_3\text{-}Fe_2O_3$) (English and Turner). $R_{O,Ca}=12.55N_{Ca}$.

being considered gives the desired difference between the constants.

For ternary or more complicated glass systems, one can make use of Eq. (13), substituting average values for N_M and $N_{M'}$.

A similar procedure can be used in changing from $a'_{M(G)}$ to $a'_{M(B)}$ constants and vice versa.

CALCULATION OF REFRACTIVE INDICES FROM COMPOSITIONS

Combining Eqs. (4) and (5), one obtains

$$n_D=1+\frac{\sum a_M N_M}{V_O}=1+\rho\sum a_M N_M\cdot\sum\frac{n_M f_M}{W_M}. \quad (19)$$

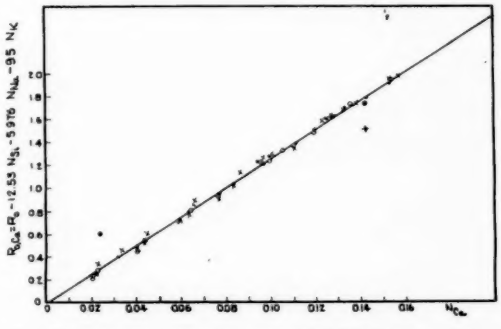

FIG. 6. ● $Na_2O\text{-}CaO\text{-}SiO_2$; × $K_2O\text{-}CaO\text{-}SiO_2$; ○ $Na_2O\text{-}K_2O\text{-}CaO\text{-}SiO_2$ (Peddle). $R_{O,Ca}=12.55\ N_{Ca}$.

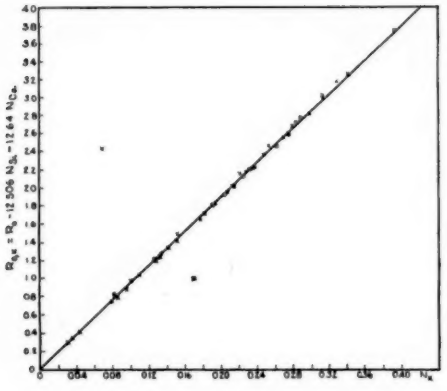

FIG. 7. □ K_2O-SiO_2; × K_2O-CaO-SiO_2 (Merwin and Morey). $R_{O,K}=9.53N_K$.

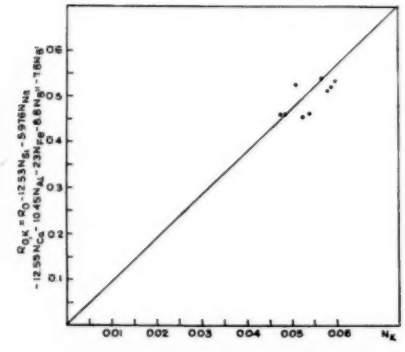

FIG. 8. Na_2O-K_2O-CaO-B_2O_3-SiO_2(-Al_2O_3-Fe_2O_3) (Turner and Winks). $R_{O,K}=9.5N_K$.

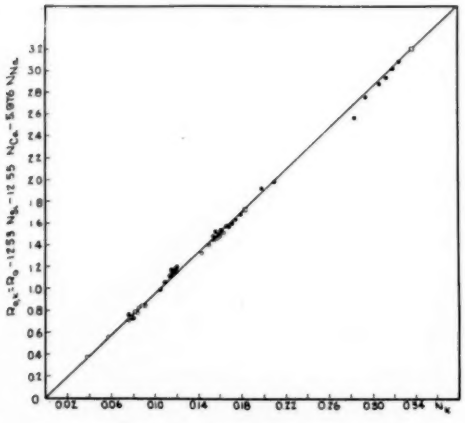

FIG. 9. □ K_2O-SiO_2; ● K_2O-CaO-SiO_2; ○ Na_2O-K_2O-CaO-SiO_2 (Peddle). $R_{O,K}=9.5N_K$.

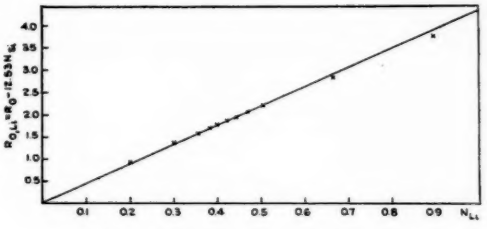

FIG. 10. Li_2O-SiO_2 (Kracek). $R_{O,Li}=4.4N_{Li}$.

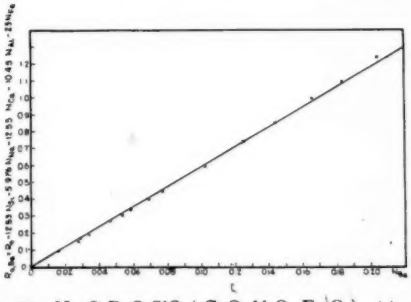

FIG. 11. Na_2O-BeO-SiO_2(-CaO-Al_2O_3-Fe_2O_3). × series *A* and V; ● series *P* (Becker). $R_{O,Be}=5.9N_{Be}$.

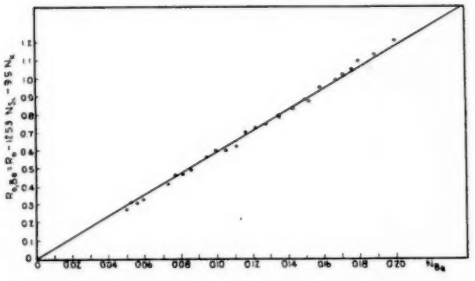

FIG. 12. ● Na_2O-BeO-SiO_2; ○ K_2O-BeO-SiO_2 (Lai and Silverman). $R_{O,Be}=5.9N_{Be}$.

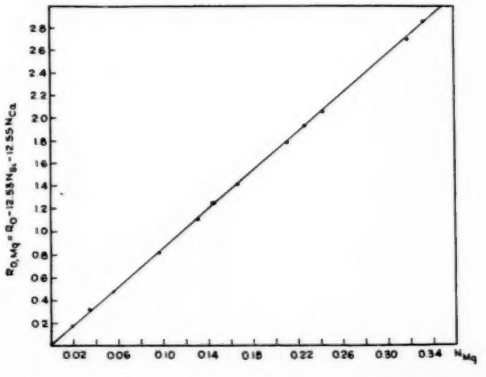

FIG. 13. MgO-CaO-SiO_2 (Larsen). $R_{O,Mg}=8.6N_{Mg}$.

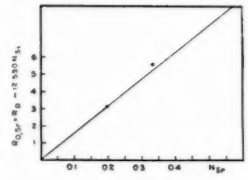

FIG. 14. $SrO\text{-}SiO_2$ (Eskola). $R_{O,Sr} = 16N_{Sr}$.

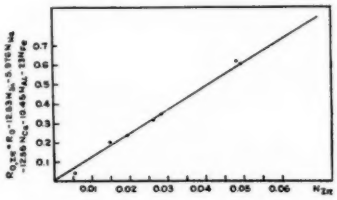

FIG. 16. $Na_2O\text{-}ZnO\text{-}SiO_2$(-CaO-$Al_2O_3$-$Fe_2O_3$) (English, Turner and Winks). $R_{O,Zn} = 12.2N_{Zn}$.

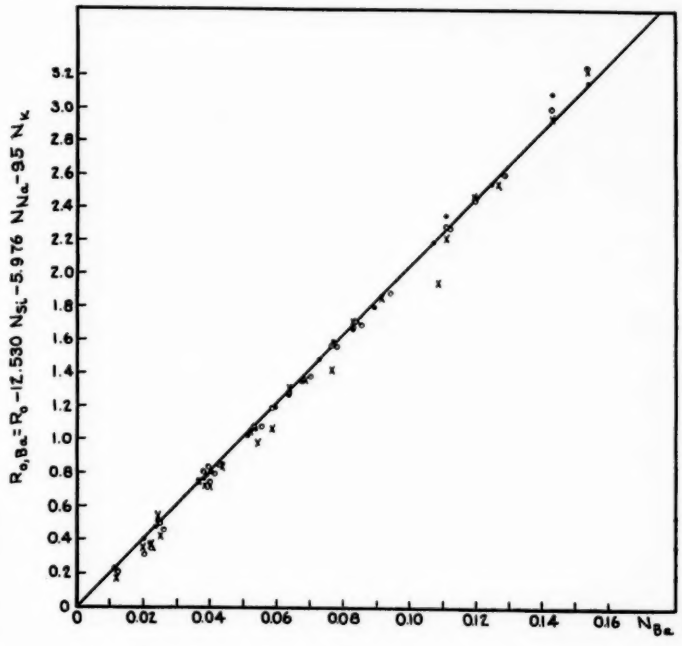

FIG. 15. ● $Na_2O\text{-}BaO\text{-}SiO_2$; ○ $K_2O\text{-}BaO\text{-}SiO_2$; × $Na_2O\text{-}K_2O\text{-}BaO\text{-}SiO_2$ (Peddle). $R_{O,Ba} = 20.5N_{Ba}$.

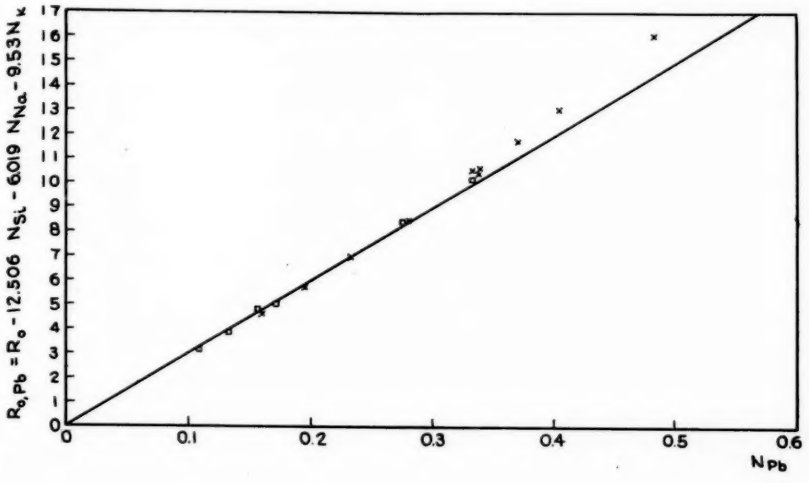

FIG. 17. × $Na_2O\text{-}PbO\text{-}SiO_2$; □ $Na_2O\text{-}K_2O\text{-}PbO\text{-}SiO_2$ (Merwin and Andersen). $R_{O,Pb} = 30N_{Pb}$ for small N_{Pb}.

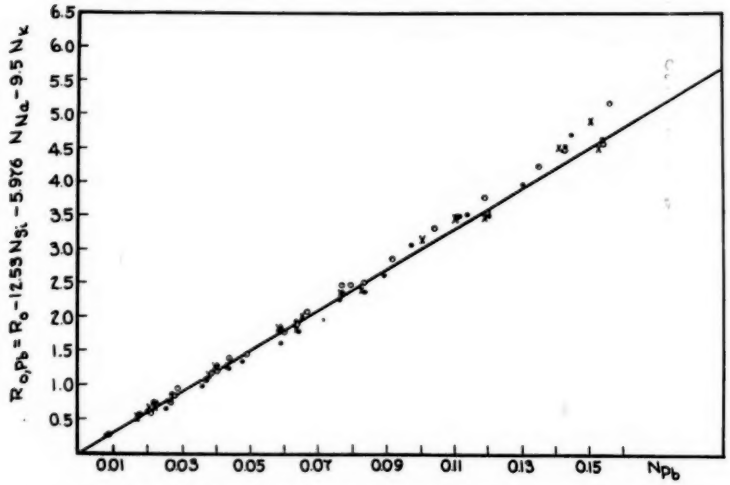

FIG. 18. Na_2O-PbO-SiO_2; ○ K_2O-PbO-SiO_2; × Na_2O-K_2O-PbO-SiO_2 (Peddle). $R_{O,Pb} \approx 30 N_{Pb}$ for small N_{Pb}.

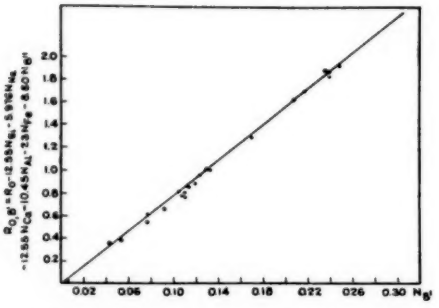

FIG. 19. ● Na_2O-B_2O_3-SiO_2(-CaO-Al_2O_3-Fe_2O_3); ○ Na_2O-K_2O-CaO-B_2O_3-SiO_2(-Al_2O_3-Fe_2O_3) (English and Turner). $R_{O,B'} = 7.8 N_{B'}$.

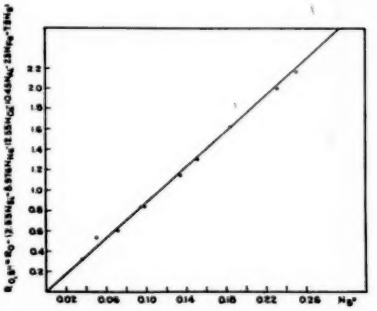

FIG. 20. Na_2O-B_2O_3-SiO_2(-CaO-Al_2O_3-Fe_2O_3) (English and Turner). $R_{O,B'} = 8.8 N_{B''}$.

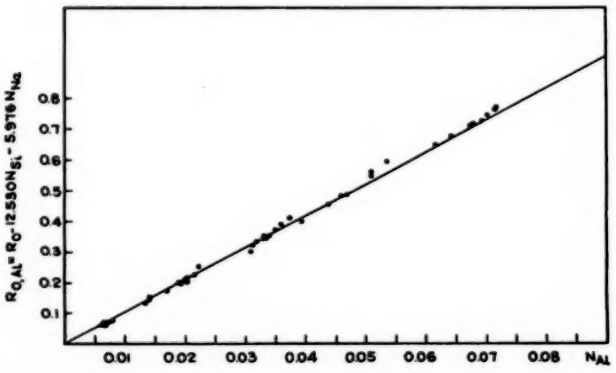

FIG. 21. Na_2O-Al_2O_3-SiO_2 (Faick, Young, Hubbard and Finn). $R_{O,Al} = 10.45 N_{Al}$.

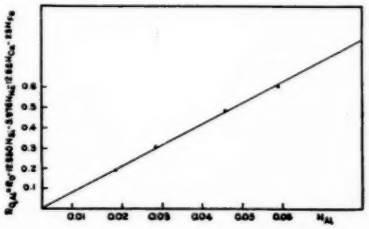

FIG. 22. Na_2O-Al_2O_3-SiO_2(-CaO-Fe_2O_3) (English and Turner). $R_{O,Al} = 10.5 N_{Al}$.

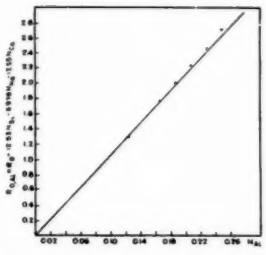

FIG. 23. Na_2O-CaO-Al_2O_3-SiO_2 (Larsen). $R_{O,Al} = 10.5N_{Al}$.

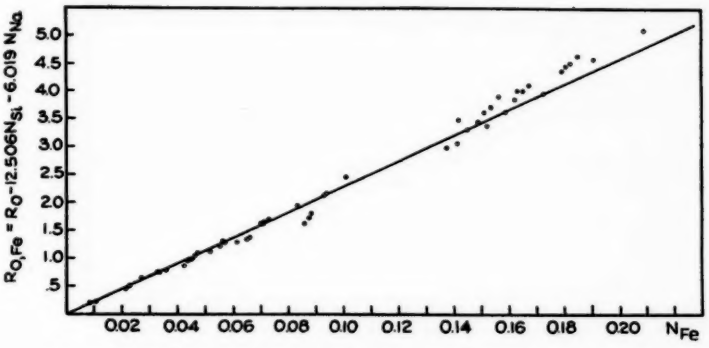

FIG. 24. Na_2O-Fe_2O_3-SiO_2 (Bowen, Schairer and Willems). $R_{O,Fe} = 23N_{Fe}$.

Similarly, from Eqs. (8) and (7),

$$n_D^2 = 1 + \frac{\sum a'_M N_M}{V_O} = 1 + \rho \sum a'_M N_M \cdot \sum \frac{n_M f_M}{W_M}. \quad (20)$$

From these equations, the refractive index, n_D, is readily calculated from the density, the composition, and the appropriate a_M or a'_M values.

If the density is unknown, one may use either the equation

$$n_D = 1 + \frac{\sum a_M N_M}{k + b_{Si} + c_{Si} N_{Si} + \sum' c_M N_M}, \quad (21)$$

or

$$n_D^2 = 1 + \frac{\sum a'_M N_M}{k + b_{Si} + c_{Si} N_{Si} + \sum' c_M N_M}, \quad (22)$$

obtained by substituting Eq. (1) into Eqs. (19) and (20). The values of the constants to be used in the denominators of these equations have previously been given.[1]

The difference between the experimental value of n_D and that which would be calculated from Eq. (19), for any of the glasses represented by a point on one of the figures of this paper, equals about one-fifteenth the deviation of the point from the straight line (since V_O is *roughly* 15 for all these glasses.) For the sodium silicates, the average magnitude of this difference is thus (see Table I) about 0.0003 or 0.0004. The average is somewhat larger in the other glass systems.

The differences between the *squares* of the observed refractive indices and the n^2 values calculated by means of Eq. (20) are likewise obtained (approximately) by dividing the deviations from the straight lines by 15. One must divide again by 3 (for an average refractive index of about 1.5) to obtain the difference between observed and calculated refractive index values. As we have seen (Table I), this averages about 0.0002 to 0.0005 for the best data in the $Na_2O - SiO_2$ system.

It should be pointed out that these deviations are due not only to deviations from additivity but also to experimental errors in density, composition, and refractive index determination.

The error in $n-1$ or n^2-1, introduced by using calculated values of the density, that is, by using Eq. (21) or (22) instead of (19) or (20), is of the order of magnitude of one-tenth of one percent or less.[1] This is little more than the probable error of most of the experimental density determinations. In many cases the calculated density (or V_O) value is doubtless more accurate than the experimentally determined value.

DISCUSSION

Table IV shows a comparison of a_M values obtained for silicate glasses with those computed from the corresponding crystalline oxides for which data are available. The two sets of values agree roughly with each other, but there are differences which are probably significant. These can be attributed to the differences in atomic arrangement (coordination number, etc.) and to the effect of silicon atoms on the polarizability of adjacent oxygen atoms.

The correspondence shown in this table would justify one in applying constants determined from measurements on the crystalline oxides to glasses, at least when the relative amount of the metallic element concerned is quite small.

It should be emphasized that the a_M and a'_M values are not atomic or ionic refractions of the elements M. If the polarizabilities (and so the refractions) of the atoms already present in a given glass were not affected by the addition of other atoms (in the ratio in which they occur in the pure oxide), each a_M or a'_M value would be the refraction per mole of the oxide M_mO_n divided by m, or, equivalently, the refraction per gram atom (or ion) of the element M plus the refraction per $n/m(=\nu/2)$ gram atoms (or ions) of oxygen. We may be sure, however, that the polarizabilities of the atoms or ions (especially oxygen) in the glass *are* appreciably affected by the addition of other components. This complicates the interpretation considerably. The fact that the simple relations given in this paper actually hold shows that the effect of the addition of another component to a glass on the refraction of the atoms already there is quite closely proportional to the relative amount of the added component.

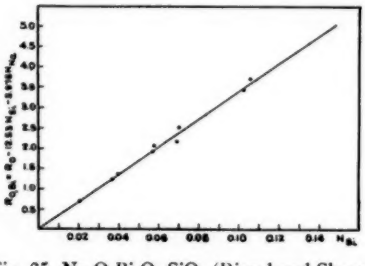

Fig. 25. Na_2O-Bi_2O_3-SiO_2 (Riegel and Sharp). $R_{O,Bi}=34N_{Bi}$.

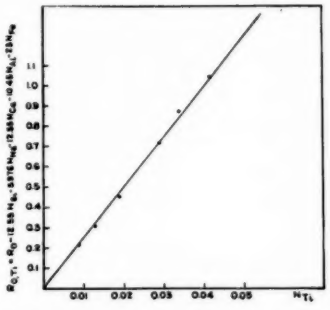

FIG. 26. Na_2O-TiO_2-SiO_2(-CaO-Al_2O_3-Fe_2O_3) (Sheen and Turner). $R_{O,Ti}=25N_{Ti}$.

A comparison of the actual values of a_M and a'_M for the different elements will not be made at this time. The subject may be discussed in a later paper.

ACKNOWLEDGMENT

Miss Dorothy Owen has performed most of the arduous computational work involved in the preparation of this paper. For her competent assistance, the writer is glad to express his indebtedness.

Research Note

Uninterrupted Measurements with a Rotating Sector*

LIGHT transmission measurements with a rotating sector are facilitated by use of a mechanism permitting a change in aperture while the sector is in motion. Kortüm[1] has described such a sector. We have, however, observed fluctuations in the radiation detecting unit (photo-cells and amplifier circuit containing an FP54 tube) as the sector was brought to rest to record the sector opening. The use of a contact stroboscope permits readings of the sector opening while the sector is in motion, and facilitates measurements considerably.

The sector was operated at 900 r.p.m.; the scale consisted of 1000 divisions on a 12″ disk; two verniers were on the second disk. It is necessary to close the stroboscope circuit at exactly the same position at every revolution in order to obtain a clear image. The circuit is closed through a pair of contacts, one of the contacts being mounted rigidly on the hub of the rotating sector. The contactor is illustrated in Fig. 1. The circuit used is illustrated in Fig. 2; it has been adapted from Germeshausen and Edgerton.[2]

This unit has been in operation for some time with quite satisfactory results.

We are indebted to Mr. Knut Wilhelmson for assistance in the design and construction of the unit.

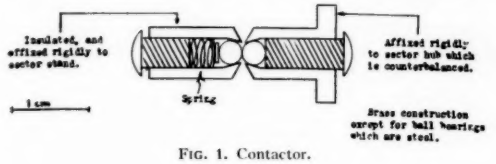
Insulated, and affixed rigidly to sector stand.
Affixed rigidly to sector hub which is counterbalanced.
Spring
1 cm
Brass construction except for ball bearings which are steel.

FIG. 1. Contactor.

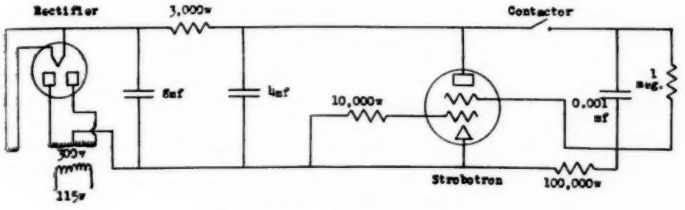

FIG. 2. Stroboscope circuit.

LOUIS HARRIS
JACK ADOLPHUS KYGER**
Massachusetts Institute of Technology, Cambridge, Massachusetts,
June 11, 1940.

[1] G. Kortüm, Zeits. f. Instrumentenk. **10**, 373 (1934).
[2] K. J. Germeshausen and H. E. Edgerton, Electronics **10**, 12 (1937).
* Contribution No. 459 from the Research Laboratory of Physical Chemistry, Massachusetts Institute of Technology, Cambridge, Massachusetts.
** Present Address, Mallinckrodt Laboratories, St. Louis, Missouri.

On the Geometry of Optical Indicators

Resumé of a paper of similar title by K. J. DeJuhasz, published in the Journal of The Franklin Institute, Vol. 229, No. 1, January, 1940, pp. 53–80, with 21 figures.

INSTRUMENTS using an optical lever for magnifying small displacements are extensively used in physics and engineering. Examples are the optical gage, extensograph, vibrograph, pressure- and force-indicator, and oscillograph. While these instruments exhibit, according to their purpose, a great diversity of design, the principles of optical magnification are common to all. This paper is a general treatment of the geometric relationships of the optical lever.

The optical lever comprises essentially: (1) the light source and the incident light beam which are, as a rule, stationary in space; (2) the mirror intercepting the incident light beam and capable of being rotated about two axes of rotation as a function of the variables to be recorded; (3) the reflected beam describing a trace on the screen which is stationary in space. Depending on the attitude of the mirror to its axes of rotation, and to the incident light beam, and on the shape and location of the screen, the trace of the reflected beam will be a curve drawn, in general, in a curvilinear coordinate system. The trace of the reflected light beam on the screen, corresponding to the rotation of the mirror about one of its axes with no rotation about the other axis is difficult to define in a general manner. The analysis, becomes possible, however, and even simple, by assuming the mirror to be located at the center of a spherical screen and capable of rotation about an axis m which is a diagonal of the sphere. The light source is assumed to be located at a point on the sphere and the incident light beam in another diagonal of the sphere, intersecting the latter also at the diagonally opposite point. The attitude of the mirror to its axis of rotation is defined by the angle enclosed by the mirror-normal with the axis m. On rotating the mirror about the axis m by 360 degrees, the mirror-normal will describe a circular cone which will intersect the sphere in circles. The reflected beam will describe, in general, two cones (which are not necessarily circular cones), the intersection of which with the spherical screen will be a curve of fourth degree. It is shown in the paper that the trace of the reflected light beam on the sphere is identical with the intersection of the sphere with a circular cone, which is similar in vertex angle and direction to the cone described by the mirror-normal, but is parallel-transferred with its vertex to the L' point (the latter being the point on the sphere diametrally opposite to the light source). This rule is called the "theorem of the transposed cone."

This theorem enables one to seek out those arrangements of the optical system which give a simple coordinate system, i.e., orthogonal or polar.

In actual optical indicators the screen is not spherical, but either plane, or such a surface (such as a cone or cylinder) which can be developed into a plane. It is shown that if the incident light beam is made to coincide with the axis of rotation of the mirror, then the reflected beam will describe a circular cone coaxial with the axis of rotation, and will produce, on a cylindrical screen coaxial with the axis of mirror rotation, an orthgonal coordinate system and, on a plane screen perpendicular to the axis of rotation, a polar coordinate system. This optical arrangement is illustrated schematically, and the embodiment of the principle in an actual design is shown, in the original paper.

In the light of this theorem a number of other optical indicators which have been built are analyzed in the paper.